United States Naval Postgraduate School

THESIS

AN INTEGRATED-CIRCUIT PIANO TUNER
FOR
THE EQUAL-TEMPERED KEYBOARD EMPLOYING
A
TUNEABLE FIXED-COEFFICIENT DIGITAL FILTER

by

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ABSTRACT

A study of the physics of the piano reveals that while the upper partials of the steel strings are the eigen-frequencies of the complex tone, they are not integer multiples of the respective fundamentals. To properly measure and tune these eigen-partials, a digital filter capable of sweeping a major portion of the audio-frequency spectrum had to be implemented. Such a filter, a tuneable fixed-coefficient digital filter, is discussed as well as a simple pole-zero design procedure for determining the required coefficients. Each module, including the Frequency Deviation Detector and Counter, the Time-Base Generator, the Digital Filter, the Reference Frequency Generator and the Display and Control Module, of the proposed tuner is illustrated and discussed.

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I. INTRODUCTION

The purpose and end result of this paper is to demonstrate how a digital piano tuner may be constructed. After only a small amount of investigation, however, it became readily apparent that a certain understanding of not only the basic physics of the piano, but also of music theory, was essential before a true definition of the problem could be established.

It was relatively simple to find many books on music and sound that gave the precise frequencies for each note of the "Well-Tempered Piano"[1] (See Fig. 1). Using these frequencies it proved no great task to design and build a digital circuit that would in fact measure, with a very high degree of accuracy, these various notes.[2] It was soon found, however, that if this was done, such tuning would produce complete aural chaos to both the trained and untrained ear alike. As asserted above, to fully appreciate this "discovery" a small amount of musical and sound theory must be presented.

A. SOUND AND PHYSICS OF THE PIANO

When any string is struck or plucked and then allowed to vibrate, it not only vibrates at its fundamental frequency but also at its various upper harmonics or partials [Refs. MF-3, MJ-10 and MO-14].

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[1] Bach established the scale of equal temperament as an accepted scale in the musical world. His pieces for piano were all written in this scale. For a discussion of the history and explanation of the scale itself, see Refs. MH-5, M)-14, and MW-24.

[2] See section head "A Frequency Deviation Detector and Counter Module."

The fundamental frequency is determined by the length of the wire or string being considered and at what place along its length it is struck [Ref. MJ-10]. The upper partials occur because of a certain amount of stiffness in almost all strings, especially the steel strings used in the piano.

Fig. 2 illustrates the vibration of an ideal string, that is, one without any stiffness. This string can be made to vibrate at many different frequencies. The fundamental frequency (a) produces a pure tone rarely heard in music. The higher-pitched partial tones, or overtones, are produced by harmonic vibrations (b) and (c), whose frequencies are integer multiples of the fundamental frequency.

Fig. 3 illustrates the simultaneous vibration of a string at two or more different frequencies and illustrates the mode that is normal for all stringed instruments.

The complex sound produced by this combination of separate tones has a timbre, or characteristic quality, that is determined largely by the number of partial tones and their relative loudness [Refs. MB-1, MC-2, MJ-10, MM-12, MP-19 and MO-14]. It is timbre that enables one to distinguish between two musical tones that have the same pitch and loudness, but are produced by two different musical instruments. With this in mind, it seems reasonable to conclude that the upper partials and not the fundamental frequency are the determining factors in the sound that is perceived [Ref. MC-2].

In order to further substantiate this conclusion, a test done by Bell Telephone at the Smithsonian Institute several years ago was investigated. The Bell engineers made two recordings of musical

instruments, opera singer, noises and everyday sounds. In one, the fundamental frequency was extracted, while the other was left untouched. When played before both laymen and professionals in different fields, the groups were unable to determine which recording had the missing fundamental! In fact, even when all the frequencies of a musical composition below 300 hertz were removed, the quality of the music remained the same to a surprising degree [Ref. MC-4]. The "case of the missing fundamental" has been attributed to the fact that the ear uses periodicity and not frequency as a basis for pitch perception [Ref. MP-18].

Another interesting fact of similar nature is brought out in Refs. MC-2, and MJ-10. They show, for the modern piano, that although the fundamental is the "loudest" when the key is first struck it rapidly dies out and upper partials, having a much longer decay time, take over the pitch of the note. (In very old pianos, the fundamen- is actually absent!) All of this further substantiates the conclusion above that the upper partials are the most important frequencies in a complex musical sound.

"So what?," may very well be the question in the mind of the reader at this time. With a seemingly logical approach, it could very well be asserted that when the fundamental is measured, the upper partials are also measured and once again only a relatively simple frequency measuring scheme is required. This is, however, not the case.

As stated before, piano strings are made of steel because of the tremendous stresses they are required to withstand in order to

be able to generate an appreciable amount of sound. This use of a "wire" or steel string, unlike the gut string of the violin, introduces a certain amount of stiffness or inelasticity. This inelasticity has the very undesirable effect on the upper partials of causing them to vibrate at frequencies that are <u>not integer multiples of the fundamental frequency</u>. No longer can the formula:

$$f_n = nf_o \tag{1}$$

be applied in determining the n^{th} partial. The formula is shown in Ref. MF-3 to be:

$$f_n = nf_o((1 + Bn^2) / (1 + B))^{1/2} \tag{2}$$

where B is dependent on the dimensions of the wire! B is in fact a function of the length squared, diameter, Young's modulus of elasticity, the area of the cross section, the radius of gyration squared and the tension of the wire when in the neutral position. As can be easily imagined this causes the upper partials to differ from piano to piano and makes completely useless any attempt to tune by zero beating the fundamentals.[3] It should be noted here that the inharmonicity caused by this string stiffness always makes the upper partials sharp with respect to the desired, pure, integer harmonic. It is less pronounced in the lower octaves and greater in the higher registers.

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[3]In Ref. MM-13, Professor Franklin Miller suggests a means whereby the desirable result of reducing this inharmonicity of partials to a negligible quantity by applying a small amount of mechanical loading near one end of the piano string.

With the above comments made, it appears appropriate to make a small digression into the field of music theory and see what effect this inharmonicity has on a musical score.

B. MUSIC THEORY

The average human ear can distinguish about 1,400 discrete frequencies. However, in the equally tempered scale covering the hearing range from 16 to 16 x 10^3 hertz, there are only 120 discrete tones.[4]

Comparatively few people are capable of recognizing the true pitch of a musical tone. A great many individuals are able, though, to distinguish the ratio of two frequencies. Furthermore, most people recognize that when two notes are sounded together or immediately following one another they either produce a pleasing effect or give a decidedly unpleasant reaction. In music theory these reactions are termed consonance and dissonance, respectively. The frequency ratios that produce these conditions are termed musical intervals or simply intervals.

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[4]Although really beyond the scope of this paper, the equal tempered scale brought to the fore-front by Bach in his pieces for the "Well Tempered Klavier" is of great importance in music theory. The scale allows the musician or composer to modulate or change keys of a composition without changing the entire frequency spectrum of the piano. This latter change in the frequency spectrum for every change in key was why the more pleasing-to-the-ear scale of just intonation was finally dropped. It is interesting to note that all music before Bach's time was written in the scale of just intonation and modern day audiences are therefore not hearing these pieces as composed because of the change in scale. For a much deeper and interesting discussion of this subject see Refs. MH-6, MJ-9, MO-14, MP-15, Mw-21 and MW-24.

The question now arises: why when two notes e.g., C4 (261.63 Hz) and D4 (293.66 Hz) are sounded together a very rough sensation is produced, while when C4 and E4 (329.63 Hz) are played, a very smooth and pleasing musical effect occurs. The answer was found first by the great investigator Helmholtz:

> When two musical tones are sounded at the same time, their united sound is generally disturbed by the beats of the upper partials (harmonics), so that a greater or less part of the whole mass of sound is broken up into pulses of tone, and the joint effect is rough. This relation is called dissonance. But there are certain determinate ratios between frequency numbers, for which this rule suffers an exception, and either no beats at all are formed, or at least only such as have so little intensity that they produce no unpleasant disturbance of the united sound. These exceptional cases are called consonances.[5]

Helmholtz also found that this "roughness" was maximized at 33 beats/sec and went as high as 132 beats/sec before its effect on musical sound became unappreciable.

Now returning to the original question, concerning the difference between C4-D4 (a musical second) and C4-E4 (a musical third), let the whole notes in Fig. 4 represent the notes played and the stars their respective upper partials.

It becomes readily apparent from the illustration that the beating of upper partials of the second produce a beat frequency well within Helmholtz's "critical area," whereas the musical third does not.

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[5]Helmholtz,H.L.F., On the Sensations of Tone, 2d Eng. ed., trans. Ellis, A., p. 179-197, Dover Publications, 1954.

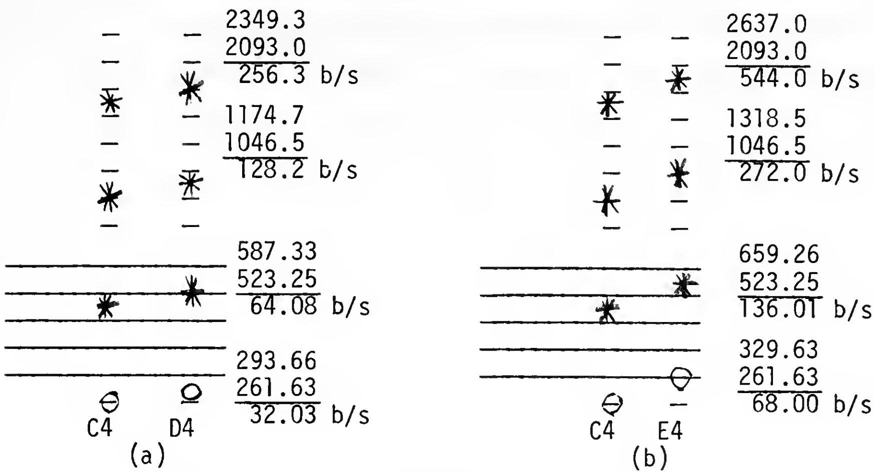

Fig. 4. Beats produced by a dissonance interval (a) and a consonance interval (b).

The intervals that produce pleasing sensations have long been known to musicians and composers. It is the mixing of these various degrees of smooth harmonious sounds by themselves and with some of the dissonance intervals that creates all of the music heard today. Table 1. gives the various consonant intervals in order of decreasing agreeable auditory sensations. Notice the appearance of the musical third, while the second is conspicuously absent.

NAME	RATIO	NAME	RATIO
Unison	1::1	Minor Third	6::5
Octave	2::1	Major Sixth	5::3
Fifth	3::2	Minor Sixth	8::5
Fourth	4::3	Major Seventh	15::8
Major Third	5::4	Minor Seventh	9::5

Table 1. Pairs of tones producing agreeable auditory effects.

Before attempting to tie all of this together, a recent investigation by Plomb [Ref. MP-18] will lay the foundation for the binding. Plomb conducted this investigation to determine what factor or factors in a complex sound caused a listener to assign it a definite pitch. The results are therefore not only applicable to the complex sounds generated by the piano but can be applied to any complex tone. Plomb found that for fundamental frequencies up to about 350 hertz, the pitch was determined by the fourth and higher partials; for frequencies up to about 700 hertz, by the third and higher partials; and for frequencies up to about 1400 hertz, the complex tone was determined by the second and higher partials. In all of the above cases, the fundamental was shown to have no bearing on the determination of the pitch once the tone had been generated. For frequencies above 1400 hertz, however, the fundamental appeared to be the determining factor. This was attributed to the fact that the ear starts having trouble detecting the periodicity of the tone at these high frequences.

It now takes little imagination to guess what would happen if the previously discussed inharmonicity of the piano wires were taken into account in the example of the second and third (see Fig. 4). The upper partials of D4 would need to be sharpened by only a very small amount before this interval would cross Helmholtz's critical area and become quite rough and unpleasant to the listener.

Thus the question arises: Just how much does the stiffness of the string cause the upper harmonics to vary? This question is easily

answered graphically in Fig. 5 [Ref. MB-1]. As is readily apparent, the higher partials can be as much as two full semi-tones sharp.[6]

C. STATEMENT OF THE PROBLEM

In the previous discussion it has been shown that while all musical strings vibrate at their fundamental frequency, they also vibrate at their various upper partials or harmonics. It was also pointed out that the upper partials and not the fundamental determines the pitch of the complex sound. This dependency on harmonics would present no real problem in the digital tuning of the piano if the strings of the piano vibrated at partials that were integer multiples of the fundamental. This, however, was shown not to be the case. Therefore in order to properly tune the steel-stringed piano, one of the upper partials, as determined by Plomb, must be filtered from the complex sound, measured and precisely tuned as if it exhibited the characteristics of a pure harmonic.

1. A Commercial Analog Device

Conn Instruments Inc. now has the only instrument on the market that attempts to solve this problem [Ref. MK-11]. It is an analog calculator and filter that will measure the upper partials of the various keys of the piano. However, Conn admits that its tuner, "The Strobotuner," requires the aid of a good pair of musical ears.[7]

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[6] A semi-tone is the difference in frequency between two adjacent keys in the equal-tempered scale. This amounts to about a six per cent difference in frequency between notes. Musicians call 1/100 of a semi-tone a cent.

[7] It is not desired to degrade from the quality of this instrument in any way. It is in fact a very fine machine that goes a long way in attempting to solve a very complex problem. Further information may be obtained by writing Conn Instruments Inc. Elkhart, Indiana 46514.

This author believes this is due to the fact that the bearing is laid by measuring the fundamentals of the octave chosen and then using nothing higher than the first partial in all subsequent tuning.[8] As Plomb's investigation has already shown, this is by no means sufficient for the majority of the notes on the piano.

This writer decided that a better tuner could be built if one would filter, measure and tune the partials that Plomb's investigation proved to be of prime importance.

2. <u>The Author's Proposed Implementation</u>

The original goal was the design of a <u>digital</u> piano tuner. This goal resulted in the decision that some sort of digital filter had to be implemented. Although there has been a great deal of work done in the field of digital filtering [Refs. EB-4, EC-7 thru EC-10, EG-15, EG-17 thru EK-19, EK-21, EM-28, EN-30, ER-33, ER-37, ER-38, ET-42 thru EV-46, EW-48 and EW-49], the area of tuneable fixed-coefficient digital filtering has been seemingly completely untouched.

Since the filtering requirements presented by the piano necessitated a filter that could sweep the frequency spectrum of the piano, it was decided to investigate this area of tuneable digital filtering. Although the remainder of this paper deals with the design of the proposed tuner, the stress was laid on the theory and implementation of this filter.

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[8] "Laying the bearing" refers to the tuning of one octave, usually the one below A4 (i.e., C3-C4), by measuring its fundamentals very precisely and then using their upper partials for all subsequent tuning. This usually results in less than perfect results. See Refs. MH-5, MW-22 and MW-25.

II. DIGITAL FILTERING

Digital filtering is the process of spectrum shaping using items of digital hardware as the basic building blocks. Thus the aims of digital filtering are the same as those of continuous filtering, but the physical realization is different. Linear continuous filter theory is based on the mathematics of linear differential equations; linear digital filter theory is based on the mathematics of linear difference equations.

An m^{th} order linear difference equation may be written as

$$y(nT) = \sum_{i=0}^{r} L_i x(nT - iT) - \sum_{i=1}^{m} K_i y(nT - iT) \qquad (3)$$

This form emphasizes the iterative nature of the difference equation; given the m previous values of the output y and the r + 1 most recent values of the input x, the new output may be computed from (3). Physically, the input numbers are samples of a continuous waveform and real-time digital filtering consists of performing the iteration of (3) for each arrival of a new input sample. Design of the filter consists of finding the constants K_i and L_i to fulfill a given filtering requirement. Real-time filtering implies that the execution time of the "computer program" for computing the right side of (3) is less than T, the sampling interval.

See Fig. 6 for a pictorial representation of (3) consisting of unit delays of time T, adders and multipliers.

It will be assumed in all further calculations that the set of input and output samples x(nT) and y(nT) are zero for all values of n less than zero.

A. THE Z-TRANSFORM

The z-transform of a sequence x(nT) is defined as

$$X(Z) = \sum_{n=0}^{\infty} x(nT)Z^{-n} \tag{4}$$

where Z^{-1} is the unit delay operator defined in the s domain as

$$Z = e^{st} \tag{5}$$

where $s = \sigma + j\omega$.

For many sequences, the infinite sum of (4) can be expressed in closed form. For example, the z-transform of the sequence

$$x(nT) = 0 \quad \text{for } n<0 \tag{6}$$

$$x(nT) = 1 \quad \text{for } n\geq 0 \tag{7}$$

is

$$X(Z) = \sum_{n=0}^{\infty} Z^{-n} = 1/(1-Z^{-1}) \tag{8}$$

The transform variable Z is, in general, a complex variable and X(Z) is therefore a function of a complex variable.

The transfer function of the filter may now be written as

$$H(Z) = Z(\text{output})/Z(\text{input}) \tag{9}$$

$$= \sum_{n=0}^{m} K_n Z^{-n} / \left(1 + \sum_{n=1}^{r} L_n Z^{-n}\right) \tag{10}$$

The coefficients of the Z^{-n} terms correspond to the value of the weighting sequence at t = nT, where n is an integer. This transfer function, in order to be physically realizable, must not contain any positive power in Z. A positive power would indicate a prediction or simply that the output signal precedes the input. This condition

implies that $m \geq r$. When $m = r$, as in this project, L_o must not be zero. In order to ensure this condition is met, L_o has been set equal to one in (10).

As asserted before, the problem is now one of finding the proper coefficients L_m and K_n.

B. CALCULATION OF THE WEIGHTING COEFFICIENTS

In the z-domain the zeroes may be written as

$$Z_o = R_o e^{j\omega T} \tag{11}$$

and the poles as

$$Z_p = R_p e^{j\omega T} \tag{12}$$

and the recursive filter transfer function may then be written as:

$$H(Z) = \frac{(1-Z_{o1}Z^{-1})\,(1-Z_{o2}Z^{-1})---(1-Z_{or}Z^{-1})}{(1-Z_{p1}Z^{-1})\,(1-Z_{p2}Z^{-1})---(1-Z_{pm}Z^{-1})} \tag{13}$$

As stated above both r and m have been set equal to n in order to meet the conditions for realizability.

There remains now the step of multiplying out (13) and matching the derived coefficients with those of (10). The desired coefficients are:

$$K_1 = Z_{o1} + Z_{02} + Z_{03} + --- + Z_{on} \tag{14}$$

$$K_k = \text{sum of the products of the } Z_{on}\text{'s taken k at a time.} \tag{15}$$

$$K_n = Z_{o1}Z_{o2}Z_{o3}---Z_{on} \tag{16}$$

and

$$L_1 = Z_{p1} + Z_{p2} + Z_{p3} + --- Z_{pn} \tag{17}$$

$$L_k = \text{sum of the products of the } Z_{pn}\text{'s taken k at a time.} \tag{18}$$

$$L_n = Z_{o1}Z_{o2}Z_{o3}---Z_{on} \tag{19}$$

Due to the binomial coefficient nature of these terms and due to the fact that all of the terms are complex variables, a Fortran program was written to determine the various L's and K's for n as high as 50. Both the program and the calculated coefficients may be found in the computer section of this work.

The frequency response of the designed filter may be obtained from:

$$H(\omega) = \sum_{n=0}^{m} K_n e^{-j\omega T} \Big/ \Big(1 + \sum_{n=0}^{r} L_n e^{-j\omega T}\Big) \tag{20}$$

This equation is also implemented in the program mentioned above.

In selecting the required poles and zeroes, two apparent limitations had to be imposed:

1. The poles and zeroes had to be real or complex conjugates in order that the desired coefficients would have real values.
2. The poles had to lie within the unit circle to produce a stable filter.

From the theory of sampled-data systems [Refs. EK-20, EM-27 and ER-34] it can be shown that the pole-zero configuration of Fig. 7, when implemented, results in a passband with a center frequency determined by

$$f_c = \theta_c / (2\pi T) \tag{21}$$

The first factor to be determined is the distance at which to place the poles and zeroes around the center angle θ_c in order to obtain the desired response. A complete investigation of this problem was made by Mooney in Ref. EM-28; the results will only be briefly summarized here.

Mooney found that close spacing of the poles and zeroes produced a sharp attenuation outside of the passband. This may easily be visualized if one thinks back on the effect the placement of poles and zeroes in the s-domain has on the sharpness of a continuous filter. It was also shown by Mooney that the passband was relatively insensitive to the location of the zeroes between the origin and the pole. However, pole placement was critical and for the proper response should be placed as illustrated in Fig. 7. For graphs and drawings of how different placement affects the response of the designed filter see the cited reference.

C. A TUNEABLE FILTER

This author reasoned that since

$$\theta = \omega T \tag{22}$$

than it should immediately follow that

$$\theta_1 = \omega_1 T \tag{23}$$

$$\theta_2 = \omega_2 T \tag{24}$$

where ω_1 and ω_2 are the respective lower and upper cut-off frequencies of the passband. Subtracting (23) from (24) gave

$$\theta_2 - \theta_1 = (\omega_2 - \omega_1)T \tag{25}$$

$$\Delta\theta = \Delta\omega T \tag{26}$$

$$\Delta\omega = \Delta\theta / T \tag{27}$$

where $\Delta\omega$ could now be defined as the bandwidth of the filter.

It was then assumed that $\Delta\theta$ could be fixed, that that the poles and zeroes could be "hard-wired" in place in the z-domain.

Equation (27) would then reduce to

$$\Delta\omega = C/T \tag{28}$$

or

$$\Delta\omega = C_1 f_s = BW \tag{29}$$

where f_s is the sampling frequency, C and C_1 are constants determined by $\Delta\theta$ and BW is the bandwidth of the filter.

Following the same reasoning it was easily shown that

$$\theta_c = \omega_c T \tag{30}$$

$$\theta_c^{\,o} = 360\ f_c\ /\ f_s \tag{31}$$

where f_c is the center frequency of the filter and θ_c the associated angle in the z-domain. Once again making the angles θ_1 and θ_2 fixed in the z-domain, (31) could be written

$$f_c = C_2/T = C_3 f_s \tag{32}$$

Equations (29) and (32) are the ones of prime interest. Equation (32) illustrates that the center frequency f_c is directly related to the sampling frequency f_s. Therefore by changing f_s it should be able to cause the filter to sweep the entire frequency spectrum with a bandwidth determined by (29). However, (29) immediately illustrates the existence of the ever-present "trade-offs" encountered in design. That is as the filter is caused to sweep and tune to ever increasing values of the frequency spectrum, the bandwidth is increased. Although the "creep" is relatively small for small values of $\Delta\theta$, it must be considered.

This creeping bandwidth may be controlled in several ways. If the bandwidth requirements of the filter are not too stringent or the "Q" of the "circuit" is not required to be too large, this

widening may be neglected or taken into account when designing. This was the case in this project. Even though the upper partials of the piano are not integer multiples of the fundamental, they are somewhere in the vicinity of the pure harmonic frequency and a relatively "relaxed-Q" filter could be implemented.

Using this reasoning as a basis, $\Delta\theta$ was selected such that as the sampling frequency was increased, the filter would continue to pass only the one desired partial of the complex sound generated. This is well illustrated in the computer output from program two.

However, if the filtering requirements and "Q" are such that no amount of variance in the bandwidth can be tolerated, the problem becomes more complex and much more interesting. Fig. 8a illustrates two tuneable digital filters in cascade and separated by an inverse digital filter. The inverse filter merely changes the output of filter #1 into an analog form in order that the resultant wave may be sampled by filter #2. The shaded area of Fig. 8b depicts the spectral output of the second filter. It would be desireable to keep this interval constant as it was swept up and down the frequency spectrum.

Let δ be a constant and represent this interval. Then

$$f_2 - f_3 = \delta \tag{33}$$

$$f_3 = f_2 - \delta \tag{34}$$

but from previous discussion

$$f_3 = \theta_3 f_{s2} / 2\pi \tag{35}$$

$$f_2 = \theta_2 f_{s1} / 2\pi \tag{36}$$

substituting (37) and (36) into (35)

$$\theta_3 f_{s2} / 2\pi = \theta_2 f_{s1} / 2\pi - \delta \quad (37)$$

$$f_{s2} = 2\pi / \theta_3 (\theta_2 f_{s1}/2\pi - \delta) \quad (38)$$

which can be reduced to

$$f_{s2} = mf_{s1} - b \quad (39)$$

where

$$m = \theta_2/\theta_3 \quad (40)$$

$$b = 2\pi\delta/\theta_3 \quad (41)$$

These equations illustrate quite clearly that a linear relationship between f_{s1} and f_{s2} can be established that results in a definite and invariant bandwidth. To calculate the center frequency f_c of this passband it is clear from Fig. 8b that

$$f_c = f_2 - \delta/2 \quad (42)$$

$$= \theta_2 f_{s1}/2\pi - \delta/2 \quad (43)$$

which results in

$$f_c = af_{s1} - d \quad (44)$$

where

$$a = \theta_2/2 \quad (45)$$

$$d = {}^{\delta}/2 \quad (46)$$

Equation (44) like (39) clearly demonstrates the fact that the center frequency of the passband is a linear function of the sampling frequency f_{s1}. Therefore it is quite possible to implement a tuneable digital filter that has an invariant bandwidth by simply adding another filter with a different sampling frequency which is linearly related to f_{s1}.

The computer programs in the back of this paper were written to test the above theory of a possible tuneable fixed-coefficient digital filter. The first one takes as inputs the locations of the poles and zeroes in the complex z-domain and calculates the proper filter coefficients. This can be done for filters up to and including n = 50. Filter responses for five different sampling intervals were then plotted via the DRAW sub-routine. The second program calculates the sampling frequency required for filtering the desired partials of the fundamental frequencies of the piano. The key number, the fundamental frequency, the partial to be filtered, the maximum allowable bandwidth to filter the partial, theta, delta theta, the required sampling frequency and the resultant bandwidth of the filter are all calculated, outputed and tabulated via this program.

Not only does this output clearly illustrate the procedure involved in tuning the piano, but it also gives a good insight into the tuneable filter. That is, it demonstrates that once the coefficients of the filter have been set (i.e., theta and delta theta), the filter can be made to sweep the frequency spectrum by changing the sampling interval.

The output from program number one, however, offers the most conclusive proof that a tuneable digital filter is realizable. As explained previously the program calculates the coefficients for an n^{th} order filter. It then takes these coefficients and using them in the equations from sampled-data theory, computes the frequency response of the filter for five different sampling frequencies. The graphs on pages 46 - 51 are self explanatory and offer convincing evidence that the previously derived equations and theory for a tuneable digital filter were correct.

D. ERRORS THAT MUST BE CONSIDERED

Since in the process of tuning the designed filter, the sampling interval was allowed to move over quite a large range, care had to be taken in avoiding the well known "aliasing" or "foldover" distortion problem. This distortion results when the sampling frequency is not at least twice as high as the highest frequency component found in the total input signal. This problem can be overcome by employing wide-band sampled data filters and "pre-warping" the frequency being filtered [Ref. EG-15 and EW-48].[9] This approach was not used because of a much easier solution that resulted from the restrictive parameters of the problem.

The highest frequency from the piano that was required to be filtered and measured was the fundamental of C8 (4186.009 Hz). Two precautions were taken to insure that foldover distortion did not occur. The circuitry involved with the microphone input was designed to act as a low-pass filter with the upper cut-off frequency just above the C8 fundamental frequency. Secondly, in the design of the filter, f_s was set equal to six times the value of the highest input frequency possible from the "low-pass filter." θ_c was then calculated from (31) and "hard-wired" into the z-domain. This resulted in a safety factor of at least five over the entire frequency spectrum to be measured.

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[9]By wide-band sampled-data filter is meant a filter whose frequency range approaches half of the sampling frequency. The cited reference gives an excellent account of this type of filter and its possible implementation.

The angle θ_c was computed as being equal to 30^0. $\Delta\theta$ was made equal to 9^0 in order to make the bandwidth of the filter as wide as possible and yet still keep it within the maximum allowable range. Ten coefficients were called for in order that the filter would have as sharp a response as possible but yet still steer clear of a delay and coefficient accuracy problem caused by the use of too many coefficients. The reader is referred to the computer outputs previously mentioned for the tabulation of all coefficients. With the mention of coefficient accuracy it seems appropriate to say something about round-off errors, word length requirements, coefficient accuracy and associated problems.

When a digital filter is realized with a digital arithmetic element, as is the case of the proposed tuner under study, additional considerations are necessary to describe the performance of the filter. There are three obvious degradations:

1. quantization of the input,
2. quantization of the coefficients of the difference equation,
3. quantization of the results of the computations.

All three types have been thoroughly investigated in Refs. EK-21, EM-27, ET-44, EK-19 and ET-45, and only the various results that affect this problem will be presented here.

The two prime considerations that enter into the selection of the input quantization size, q, are a minimum detectable level of the signal, x_{TH}, and a saturation level, x_{SAT}. From Ref. EW-48 it was found that

$$q = x_{TH}/x_{SAT} \; . \tag{47}$$

The A/D converter is then required to have a minimum accuracy of N bits, which is determined by:

$$2^{-N} = q \tag{48}$$

$$N = \log_2 (x_{SAT}/x_{TH}). \tag{49}$$

One word time is then defined by N data-bit times plus one sign-bit time. The speed of the converter, although no problem in this proposed design, may be calculated from the following formula where T = sampling interval

$$S = (2/T)\ (1 + \log_2(x_{SAT}/x_{TH})). \tag{50}$$

The third quantization problem mentioned is usually set equal to the quantization of the input and was done so in this design. See Ref. EW-48 for further amplification.

The second type of quantization error listed appears to be the most important one to this writer. It is quite easy to understand and a thorough investigation is conducted by Knowles and Olyacato in Ref. EK-19. This quantization of the coefficients changes them slightly, resulting in a new, slightly different filter response. This happens only once when the filter is first designed. One important aspect of this problem is that the more coefficients that are used, the more accurate they must be and therefore the more this error enters into the problem. This is why a filter of the order of only n = 10 was chosen. Kaiser, in Ref. EK-21, also brings out the fact that as the sampling frequency is raised the coefficients must become closer and closer to the "ideal" ones; that is, ones that would require an "infinite" number of bits for their implementation and storage. This

was the reason that the sampling frequency was set to be only six times the value of the highest input frequency instead of ten to twelve times as is done in most sampled-data networks.

It should be pointed out here that, although the sampling frequency was kept as low as possible and the order of the filter was kept relatively small, a great deal of difficulty was encountered with the coefficient accuracy problem.

The coefficients given in the computer output section were calculated using complex double-precision arithmetic on the IBM/360/67. These same numbers were obtained by two completely different algorithms so there is little doubt that they are correct. However, when the coefficients were used to obtain the frequency response of the desired filter, the undesirable effects shown in Graphs 1 and 2 in Appendix A were obtained. This was, however, not necessarily due to inaccurate coefficients. Two possible inaccuracies could have been generated by the nature of the library sub-programs used. First, in order to take the real part of the complex coefficients, the precision had to be reduced from double to single. This naturally involved some type of round-off error. This necessitated change in precision resulted in the use of the single-precision sine and cosine library tables and could have introduced another round-off error. Short of actual implementation of these coefficients in a digital filter, any further attempt to obtain the frequence response by this method was impossible.

In order to bypass this problem Equation (13) was programmed. As is readily apparent by referring back to (13), this equation greatly reduces the probability of round-off errors. This reduction is caused by the knowledge of the precise locations of the poles

and zeroes and the requirement of having to introduce only one phasor calculation. This is unlike the previous example where the order of the filter determined how many different phasors had to be solved. The results illustrated in Graphs 3-6 in Appendix A conform to all theory that has been presented. With this discussion, this author feels that if the tabulated coefficients were actually implemented in a digital filter the frequency response of Graphs 3-6 would be realized.

III. A PROPOSED DESIGN FOR A DIGITAL PIANO TUNER

A. INTRODUCTION AND OVER-ALL VIEW OF THE MACHINE

The third and final section of this paper is devoted to the actual implementation of the proposed tuner. Fig. 9 illustrates the general flow of signals and information in block diagram form. In actual operation the key to be tuned[10] is selected on the front of the control panel by the rotation of a 12-position switch representing the twelve notes of one of the equal-tempered octaves. Another rotary switch is used to select the appropriate frequency output of the Reference Frequency Generator Module (RFGM) as will be explained later. The RFGM is nothing more than a very stable, crystal-controlled I.C. pulse-train generator. The output frequency of this module had to be at least four times as high as the desired frequency to be measured. This was required in order that the intervals generated by the ring counter would be quarter periods and the subsequent error count could therefore be read in cycles.

The Time-Base Generator Module (TBGM) was added to provide the error count readout directly in hertz. This module acts as a real—time clock (RTC) and was provided with the capability of producing timing pulses every 5 or 10 seconds.

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[10]Actually the use of the word "key" here is a misnomer. All keys have more than one string in order to increase their output volume. The normal modern-day piano has, starting at the upper octaves and working towards the extreme bass, 60 keys with three strings, 18 keys with two strings and 10 keys with one string. Some of the larger concert pianos will have more but all pianos have the same 88 keys. The different strings are each tuned separately. This is achieved by inserting a tuning wedge between the strings in such a way that only one is allowed to vibrate. Extreme care must be taken that the multiple-stringed keys are tuned correctly if undesirable "beats" are to be avoided.

The Sign-Detection and Pulse-Shaping Module tests the left-most bit of the output register of the digital filter and when a positive-going change occurs, that is, when the bit changes from a "0" to a "1" a "one-shot" is triggered. Since an output pulse from the one-shot occurs only when the filtered partial to be measured crosses the "zero axis," the pulse is in synchronization with the unknown frequency of the filtered partial. This pulse from the one-shot is fed into the Frequency Deviation Detector and Counter Module (FDDCM).

The Frequency Deviation Detector and Counter Module is a logic circuit that is capable of measuring the beat frequency between the reference frequency and the unknown input. This beat frequency or error count is displayed on the control panel by a series of digital display lights. The FDDCM also determines whether or not the beat frequency is sharp (fast) or flat (slow) with respect to the reference. This enables the tuner to know in which direction to turn the tuning pin in the piano. For the laymen, this sharp-flat meter would be all that would be required for successful tuning. However, most professional tuners like to sharpen or "stretch" the upper octaves by about five cents per octave to add a certain amount of "brightness" to the sound generated [Ref. MH-8]. This requires some form of exact frequency measurement, which the FDDCM does to a very precise degree.

As mentioned previously, the input microphone and associated circuitry was designed to act as a low-pass analog filter with an upper cut-off point at approximately 5 KHz.

The Digital Filter Module multiplies the sampled input by the appropriate coefficients. This involves the implementation of shift registers and some associated logic.

The Display and Control Module (DCM) houses the various selection switches and lights for display. Four 10-counters were properly implemented and decoded to display the error in the incoming frequency from the piano.

B. THE POWER SUPPLY

The power supply had to be designed to provide more than an ampere of current at 3.6 volts DC to the various logic chips. This had to be supplied with a very low ripple factor in order to prevent confusion between the ripple and the actual timing pulses employed in the circuit. It had to also provide other lower-current supplies for additional circuitry at +6, -6, and +12 volts DC. In addition to the various bias requirements it had to meet, it was also desirable to produce a 60-Hz signal for the Time-Base Generator Module and an AC voltage to the four digital-display lights.

In order to meet all of these requirements from a single transformer, eleven diodes had to be used. See Fig. 10 for the schematic of this module.

The +12-volt supply was obtained from a voltage doubler consisting of D1, d2, C1 and C2. The full-wave rectifier made up of D3, D4 and C5 provided the -6 volts, while a second full-wave rectifier, consisting of D5 and D6 fulfilled the +6 volts bias requirement. The +6 was reduced by D9, D10 and D11 to provide the +3.6 volts required by the I. C. chips.

C. THE ANALOG TO DIGITAL CONVERTER

The A/D module is illustrated in block diagram form in Fig. 11. The operation of the ramp converter is initiated by means of a sample

gate pulse which is applied to the output of the ramp generator. The ramp generator produces the triangular waveshape shown in Fig. 12, waveform A. As this rising voltage becomes equal to or greater than the zero reference applied to comparator No. 1, a pulse is produced which is used to set a control flip-flop. When the rising voltage becomes equal to or greater than the analog voltage applied to comparator No. 2, a pulse is produced to reset the control flip-flop. The result of this operation is the production of a control pulse, the duration of which is proportional to the amplitude of the input analog signal. This control pulse is then used to gate clock pulses (which are a periodic train of timing pulses) into a counter which starts at a count of zero. The number of pulses allowed to enter the counter is proportional to the pulse width of the control pulse. Therefore, the final content of the counter is proportional to the amplitude of the analog signal. After the ramp voltage reaches a maximum, it is returned to its initial value. The trailing edge of the ramp is used to generate a reset pulse which reads out the contents of the counter into an output register and resets the counter to zero. All of the associated waveforms are illustrated in Fig. 12. The "transfer" occurs when the seven-bit counter is read-out into the read-out gates.

The seven-bit counter illustrated in Fig. 11 had in reality eight-bits. The left-most bit, the sign-bit, was dropped from the above discussion for the sake of clarity. The seven-bit counter (including the eighth-sign-bit) was implemented because it produced a very high and desirable degree of accuracy. That is, it was capable of dividing the input signal into 628 levels ($2^7 = 628$).

D. THE FREQUENCY DEVIATION DETECTOR AND COUNTER MODULE

If two frequencies are superimposed on an oscilloscope screen, the relation between them can be determined by counting the number of times they move in and out of synchronism. In reality this nothing more than determines a beat frequency, which will give the deviation of the second from the first, if the first is considered to be the reference. In existing analog systems the attempt to realize this beat frequency at low frequencies and with small errors can be quite complex and cumbersome. For example, an error of 0.01% at 400 Hz would produce a beat of one twenty-fifth of a cycle per second. Although accuracy this high is not demanded in the tuning of the piano, a reasonable degree of accuracy is required and can be realized with a saving in size, cost and required skills of an operator by the use of digital circuits. Refer to Fig. 13 for the logic diagram of the designed FDDCM.

The crystal-controlled reference frequency from the RFGM is fed into the phase generator, a simple four-stage ring counter (See Fig. 14), to produce the required timing intervals. Four intervals, A thru D, were provided to allow for the polarity of the errors to be calculated. The one-shot (mono-stable multivibrator) produces a pulse that is in synchronization, as explained previously, with the unknown frequency to be measured. This pulse, designated f_x, was ANDed separately with each of the four phases from the ring counter. This gives the output of the coincidence detector as (AF, BF, CF and DF). These signals provided the basis for the remaining logic. The error-count detector was an RS flip-flop which produced an error count whenever the unknown had cycled from coincidence with phase A to phase C. The recording of

an error count occurred only when the output of this RS flip-flop changed from "0" to the "1" state. Note that repeated pulses on the set input can produce no further error counts.

The polarity, that is whether or not the unknown is fast or slow compared to the reference, was determined by the remaining logic. Figs. 15, 16 and 17 illustrate the various waveshapes for a "sharp," "flat" and perfectly tuned unknown input. If the input is perfectly "tuned" to the reference it will appear to remain stationary when plotted against the reference with respect to time. However, if it were sharp or flat, it would tend to drift to the right or left, respectively, and thus cause an error count to be generated.

Once again, only off-the-shelf Fairchild Semi-Conductor Micro-Logic Chips were employed in all design work.

E. THE REFERENCE FREQUENCY GENERATOR MODULE

The RFGM works on the same basic principle as the pitch reference found in Ref. EL-22. That is, it takes a set, crystal-controlled frequency and uses it to drive a chain of JK flip-flops. This chain divides the crystal frequency output into the desired reference frequency outputs.

The power supply used in the cited reference was deleted in favor of the one already discussed. This was only for the sake of simplicity and in an attempt to avoid needless repetition of components. The speaker and associated circuitry were also dropped for the same reasons.

The basic block diagram of the circuit appears in Fig. 18. When one of the crystal oscillators is switched on, the output wave is taken by the input of the Logical Schmitt Trigger, illustrated in Fig. 23,

and made into a square wave of the same frequency. Take for example, the switch S_1 as being in the position indicated in Fig. 19. Then the output of the Schmitt Trigger is a 34.29 MHz square wave and is fed into the chain of JK flip-flops. With the switch in this position there are 12 flip-flops in this chain when tuning octaves five, six and seven. With no feedback, the chain will divide by 4096 ($2^{12} = 4096$). The resulting output frequency of the chain would therefore be 8371.9 Hz. Notice that this frequency is four times the frequency of the desired fundamental of C7, the desired first partial of C6 and the third partial of C5. It is therefore the proper reference frequency required to input to the FDDCM when tuning these notes.

Now if the frequency output from the previous example were buffered and fed back into the outputs of flip-flops one, two, five and six, the chain would now divide by 3866. This is because extra counts exactly equal to the difference between the desired divisor of 3866 and 4096 have been added. In arithmetic form the preceeding is illustrated by

$$4096 - 3866 = 130$$

$$2^1 + 2^2 + 2^4 + 2^5 + 2^6 = 130$$

When this division by 3866 is carried out on the input frequency of 34.29 MHz, the resulting output was calculated to be 8869.6 Hz. This is four times the desired fundamental of C7#, the desired first partial of C6# and the desired third partial of C5#. Once again the required

reference frequency to tune these notes has been obtained. This same procedure was carried out for the 36 notes in the upper three octaves and the results are displayed in Table 2.[11]

NOTE	DIVISION RATIO	DESIRED PARTIAL FREQUENCY	REFERENCE FREQUENCY
C7-C6-C5	4096	2093.0	8371.9
C7#-C6#-C5#	3866	2217.5	8870.0
D7-D6-D5	3650	2349.3	9397.2
D7#-D6#-D5#	3444	2489.0	9956.0
E7-E6-E5	3250	2637.0	10548.0
F7-F6-F5	3068	2793.8	11175.2
F7#-F6#-F5#	2896	2959.9	11839.6
G7-G6-G5	2734	3135.9	12543.6
G7#-G6#-G5#	2580	3322.4	13289.6
A7-A6-A5	2436	3520.0	14040.0
A7#-A6#-A5#	2298	3729.3	14917.2
B7-B6-B5	2170	3951.1	15804.4

TABLE 2

Reference frequency output data for the upper three octaves

As is illustrated in this table all of the upper three octaves of the piano can be tuned with the 34.29 MHz frequency setting and the chain of twelve dividing flip-flops.

As explained previously it was desired to correctly measure and tune the fourth partials of the notes in the fourth octave. This octave of frequencies can be obtained by dividing the third partials of the

[11]Although the reference frequency output is not exactly four times the value of the partial being measured, it is accurate to ± 0.5 cent, making it twice as good as the best tuning fork available.

fifth octave by two. Since it has already been shown how the RFGM is able to generate the correct frequency for the fifth octave, the only requirement is to divide this set of frequencies by two in order to obtain the correct set of frequencies for the fourth octave. That is, when tuning the fourth octave, another flip-flop must be switched in from the control panel. This is done by simply rotating the seven-position switch marked "octave" to the 4th position.

It was also desired to measure and tune the fourth partials of the lower three and one-third octaves. It would have been highly desirable to simply continue adding divide-by-two flip-flops in the dividing chain to decrease the reference frequency to its desired value. Had this been done, however, it would have been required to filter and measure the ninth partials of the third octave, the eighth partials of the second octave and the seventh partials of the first octave. According to Plomb's criterion this would have presented no problem. However, these specific partials are unfortunately suppressed by manufacturers because of the shrillness they tend to add to the music when present [Ref. EW-22].[12] Clearly, another approach was indicated.

This observation required that a second crystal with a resonant frequency of 6.7171 MHz be added. When this frequency was divided by the basic chain of twelve flip-flops, a set of frequencies was produced that was four times the desired partial frequencies of the fourth octave. Therefore, the required reference frequency had once

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[12]This is done by having the hammer hit the string between one-seventh and one-ninth of the distance from the one end of the string to the other.

again been obtained. The first and second octaves can also be tuned from this crystal by switching one more flip-flop for the second octave and two more for the first. Once again this is achieved by the rotary switch on the face of the Display and Control Module.

F. THE TIME BASE GENERATOR MODULE

The TBGM, illustrated in Fig. 19, consists of seven JK flip-flops and one dual two-input gate package (Fairchild Semiconductor RTμL 9914). Incoming 60-Hz clock pulses from the power supply are fed to FF1 and FF2 which divides the input by three. The resulting 20-Hz signal provides clock pulses at 0.05-second intervals (T = 1/f). FF3 divides the 20-Hz signal by two to provide clock pulses at 0.10-sec intervals, and FF4, FF5 and FF6 are connected in a divide-by-five circuit configuration to obtain the 0.50-second timing interval. The 2-Hz output of FF6 is divided by FF7 for the 1.0-second clock pulse. The first half of the dual two-input gate is used as an inverting amplifier to boost the divide-by-five input signal level and the other half is used to amplify the final clock-pulse output. This output chain of pulses is fed into a divide-by-ten counter illustrated in Fig. 20.

The divide-by-ten module was designed using the set and clear of the basic JK flip-flop as gates. It was able to feedback directly to the binary divider without any extra parts and inhibit counts 11 through 16. Although the circuit is quite difficult to decode, it provides a very simple, stable and cheap divide-by-ten package. The output from this circuit provides the actual five or ten-second timing pulse. When the pulse occurs it gates the FDDCM and halts all counting.

In order to ensure that the 60-Hz signal obtained from the power supply had a relatively sharp rise and fall time, a Logical Schmitt Trigger was implemented between the power supply and the input to the TBGM. It is illustrated in Fig. 21 and as shown employs a Fairchild RTμL 9914 gate. The series resistance limits the peak voltage at the gate input to two to three and one-half volts. The capacitor acts as a filter and the diode clamps the negative portion of the wave to ground.

G. THE TUNEABLE DIGITAL FILTER MODULE

The implementation of this module might have presented a real problem if the use of a tuneable filter had not proven feasible. It would have then been necessary to generate a new set of coefficients for each octave to be tuned. Fortunately, as has already been discussed in some detail, this was avoided.

The filter used has the same characteristics as the one discussed in Ref. ET-43, except that the sampling time could be varied by the tuner from the CDM. This Fixed-Coefficient Diode-Array Digital Filter was employed because of its extremely small size and simplicity of implementation. It uses the table-lookup method of product generation where the diode-array is used to store the multiplication table. A diode-array mechanization is illustrated in Fig. 23.

The input data from the A/D converter is shifted in serially to load the shift register. The contents of this register addresses the product, which is stored, in the diode array. The product is loaded into the output register which supplies the input to the next chip or array. Each chip delays the information the required interval T.

The schematic of Fig. 22 can be implemented on a single MOS chip. For this proposed tuner it would take 20 of these chips; one for each numerator and denominator coefficient. Reference ES-38 discusses recent developments in this field and also the possibility of putting many of these arrays on a single chip.

H. THE DISPLAY AND CONTROL MODULE

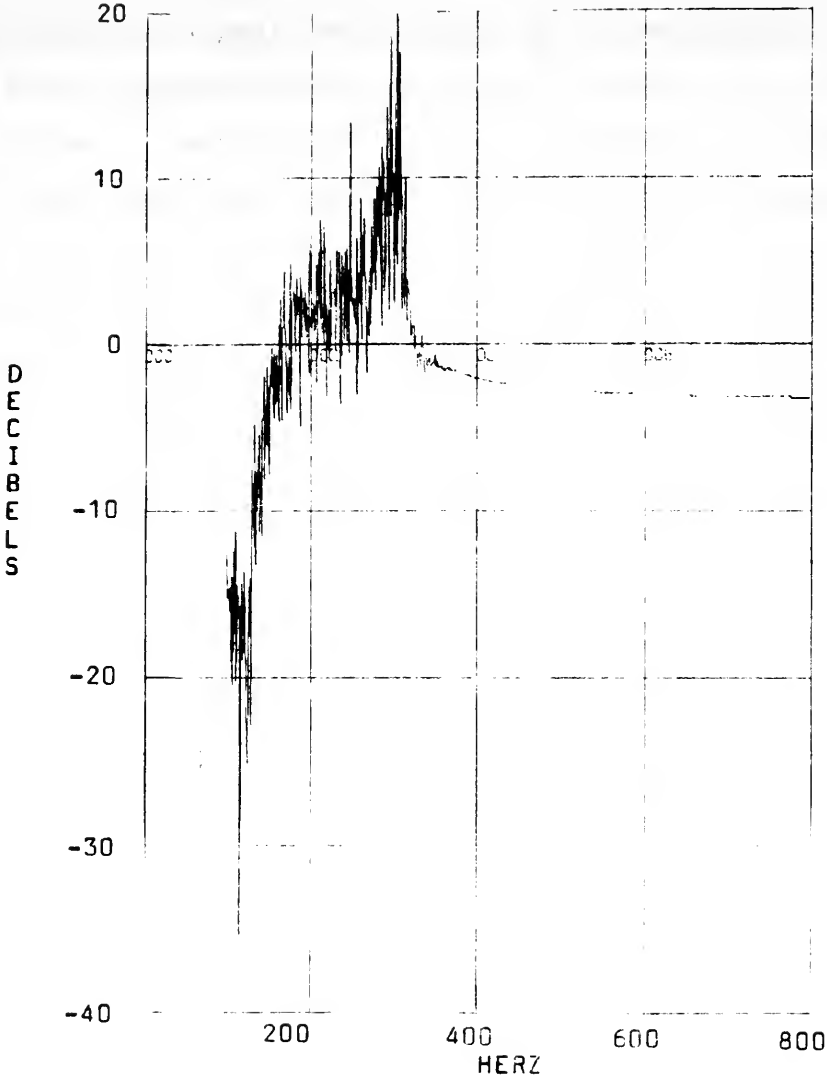
The DCM consists of a set of digital display lights, a meter indicating flat, zero and sharp, and two sets of selection switches. The selection switches control the number of flip-flops in the RFGM dividing chain and the sampling interval of the A/D. The meter is a three-terminal galvonometer and will tell the tuner whether the partial being measured is sharp or flat with respect to the reference frequency. The display lights will measure a frequency deviation up to 999.9 Hz. There are four lights on the panel and each one is used to decode a "10" counter. The counter and the required decoding scheme are illustrated in Fig. 24. The output from the first counter is used as the trigger for the second. The output from the second is used as the input for the third and the output from the third is used as the input for the fourth and final stage. Thus, a "1000" counter has been effectively achieved. When employing the usual 10 second—timing interval, the counter becomes a "100" counter that counts by tenths.

IV. CONCLUSION

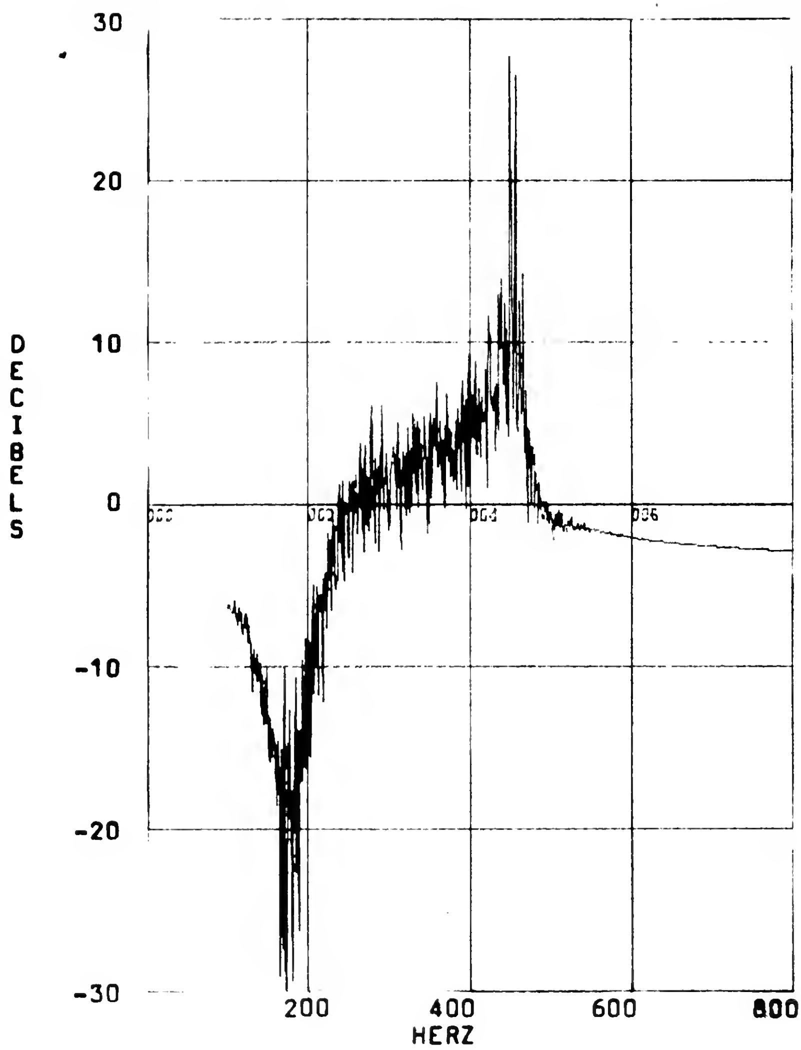
From the study of the physics of the piano, it has been shown why it was necessary to measure and tune the eigen-partials of the complex sound produced by the steel-stringed piano. It was also demonstrated how this filtering and frequency measuring could be achieved with a tuneable fixed-coefficient digital filter and several additional I.C. modules. The remaining portion of the work was then devoted to the possible implementation and construction of these modules. Based on the sound and music theory presented this proposed design seemed to offer the best electronic tuner to date. Unfortunately time did not allow for the actual interfacing of the modules and testing of the tuner as a single unit.

However, it should be stated that this author in no way advocates this machine as a panacea for the complex problem of tuning equal-tempered key-board instruments. Music and its desirable qualities are things which surpass scientific measurement because, contrary to all attempts to define them otherwise, they remain purely subjective quantities. A piano tuner who employs his well-trained ear is able to take the tastes of the period into account and tune accordingly. This machine or any other can only give him a better or more perceptive "ear."

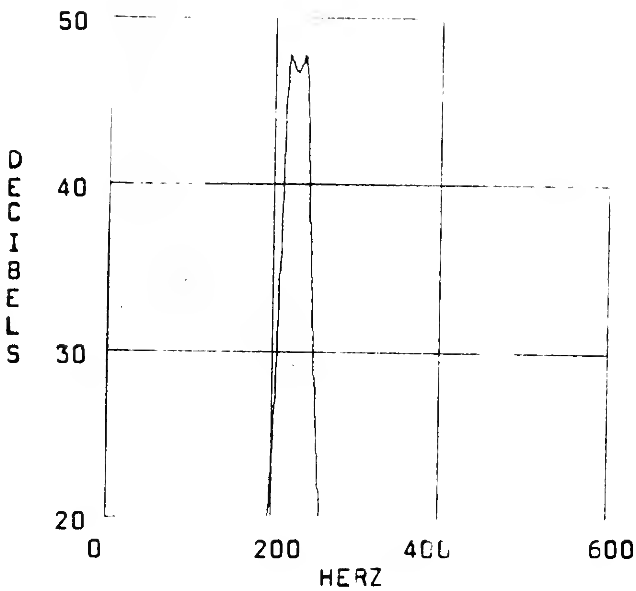
APPENDIX A GRAPHS



Graph 1. Illustrating the Results of Coefficient Inaccuracy on the Frequency Response of a Digital Filter.

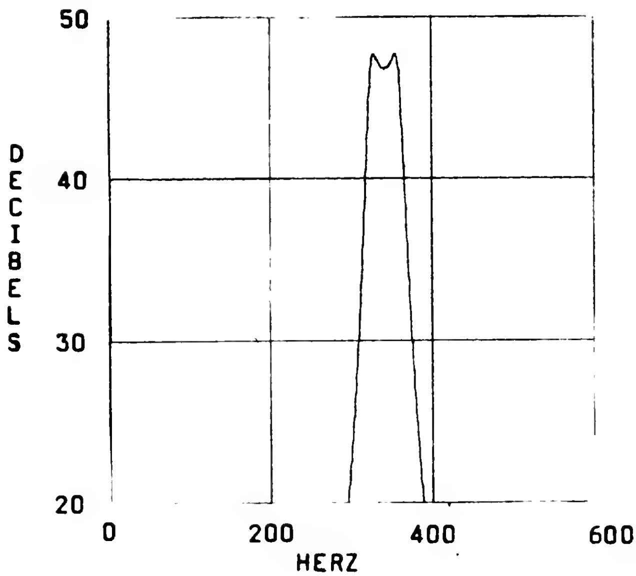


Graph 2. Illustrating the Results of Coefficient Inaccuracy on the Frequency Response of a Digital Filter.



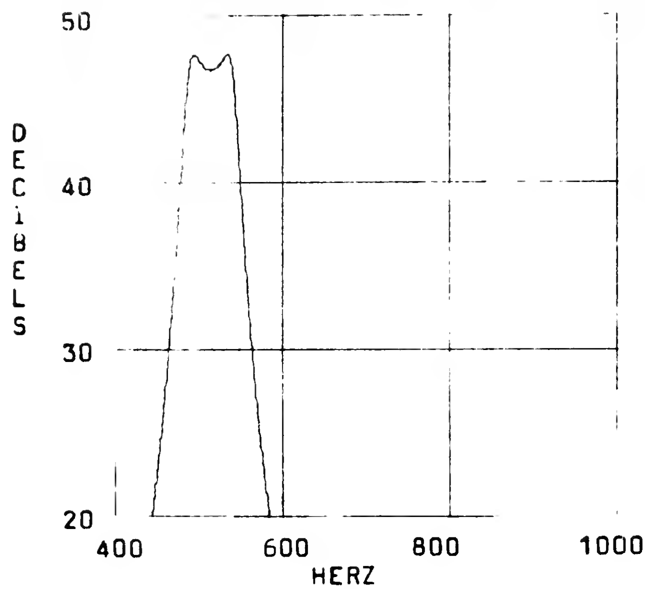
f_s=2.750 KHz n=10

Graph 3. Frequency Response of the Designed Digital Filter.



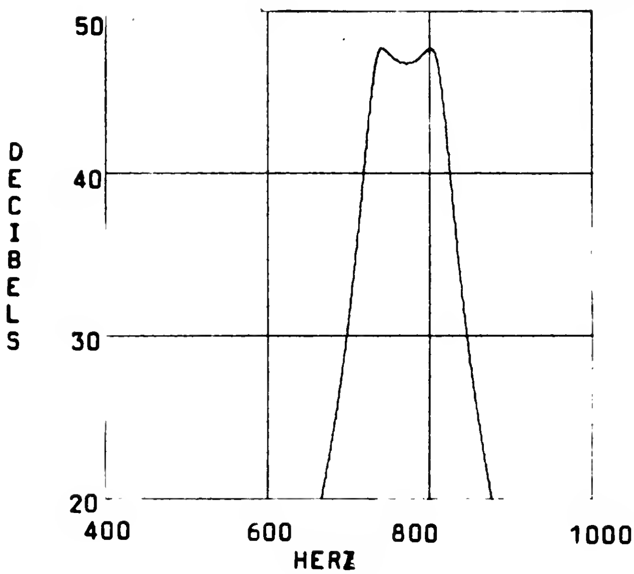
f_s=4.125 KHz n=10

Graph 4. Frequency Response of the Designed Digital Filter.



f_s=6.188 kHz n=10

Graph 5 Frequency Response of the Designed Digital Filter.



f_s=9.281 KHz   n=10

Graph 6. Frequency Response of the Designed Digital Filter.

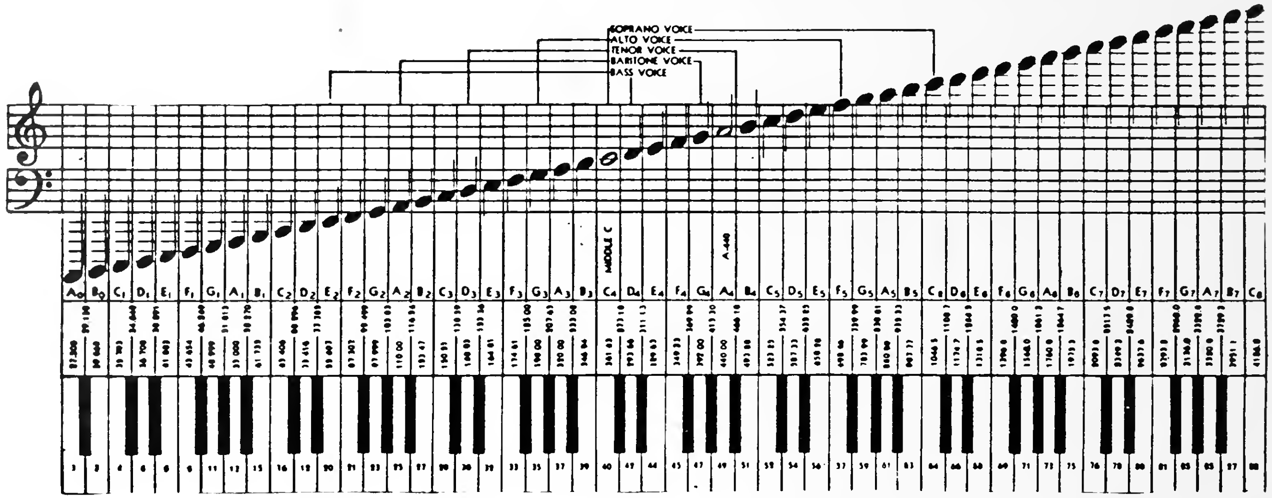


Fig. 1. Illustrating the Full Range of the Piano with the Herz of Each Note in Equal Temperament Based on the Standard Pitch of A440. (Published by permission of the Conn Instrument Company of Elkhart, Indiana.)

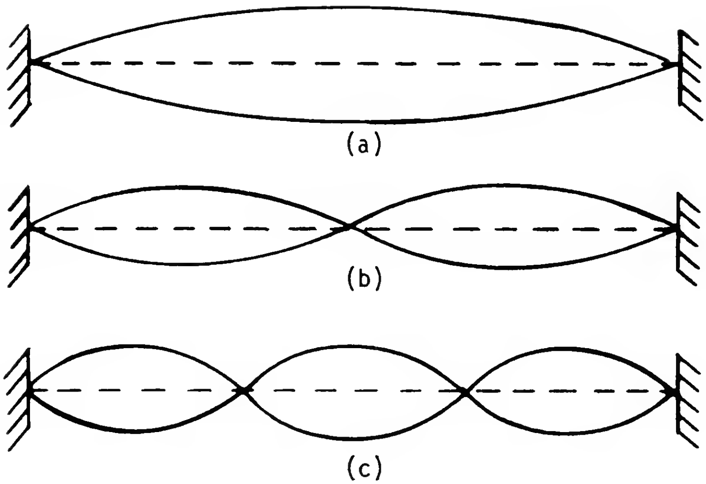


Fig. 2. Vibrations of an Ideal String.

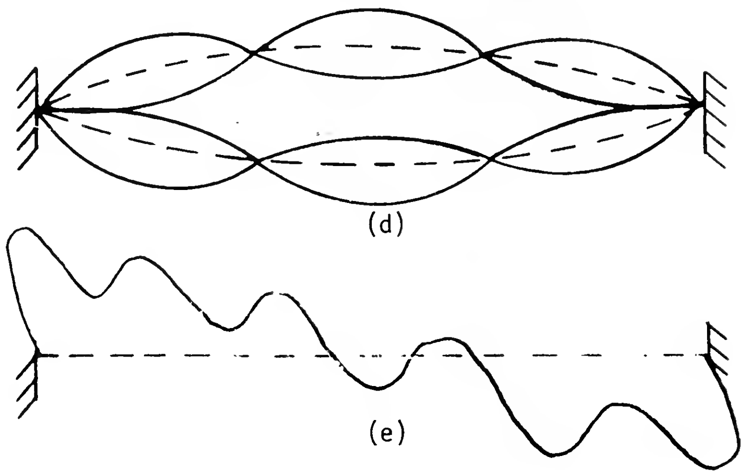


Fig 3. Typical Vibrations of a Musical String.

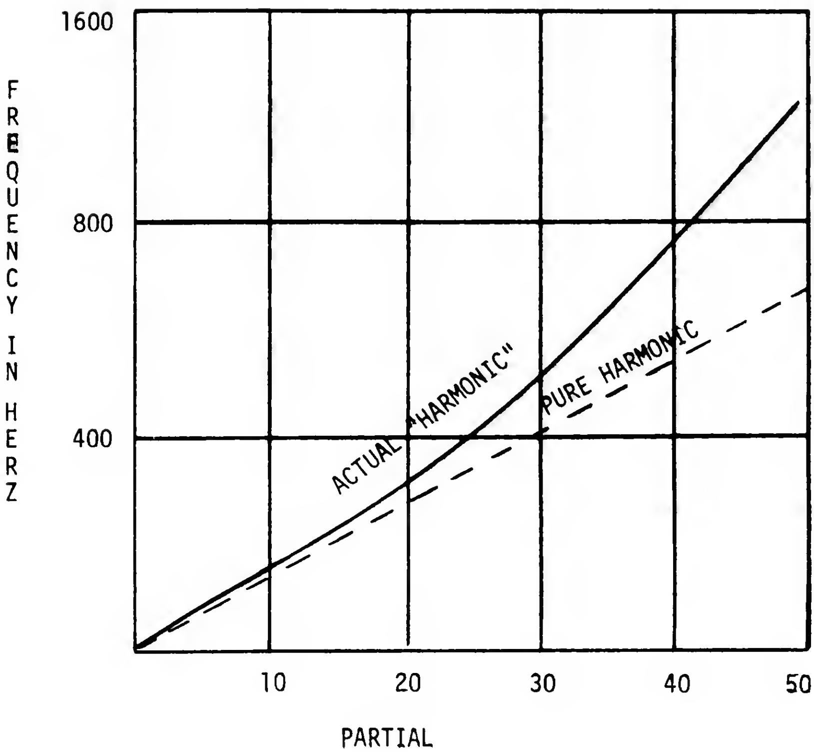


Fig. 5. Inharmonicity of the Steel-Stringed Piano

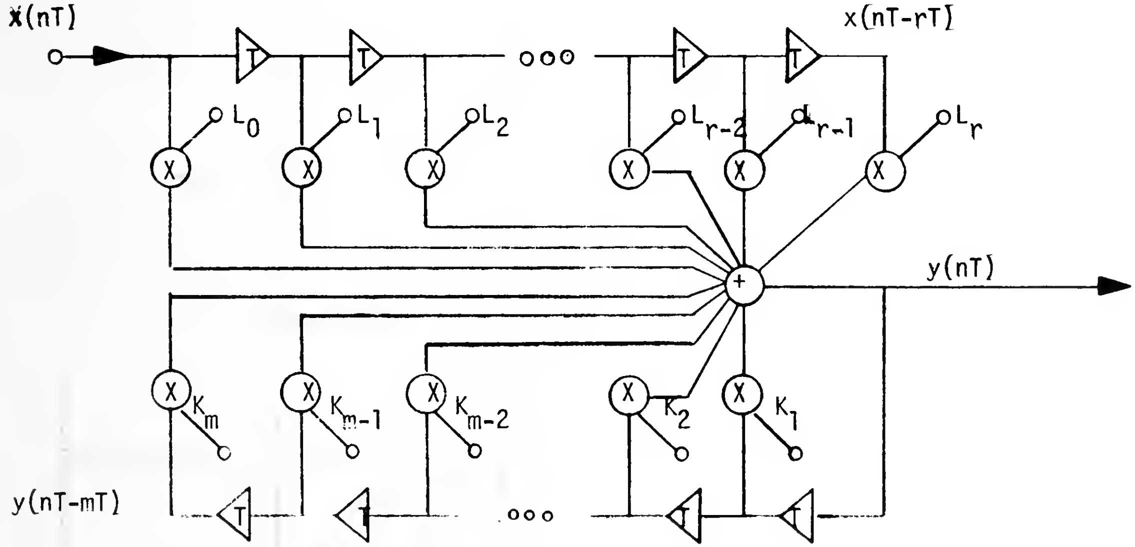
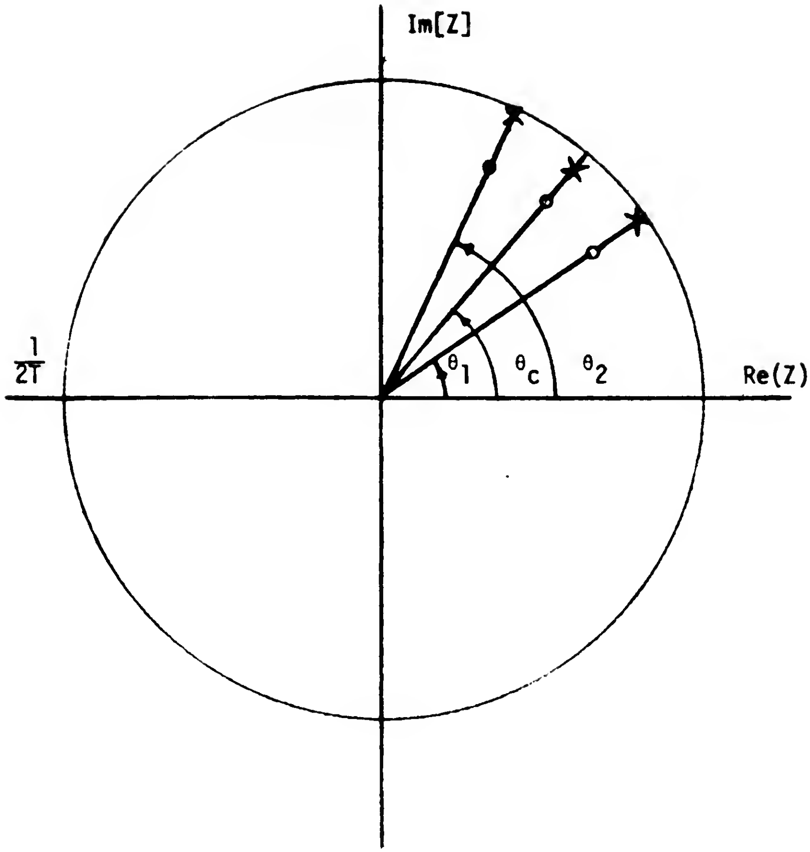


Fig. 6. Pictorial Representation of an m^{th} Order Linear Difference Equation.



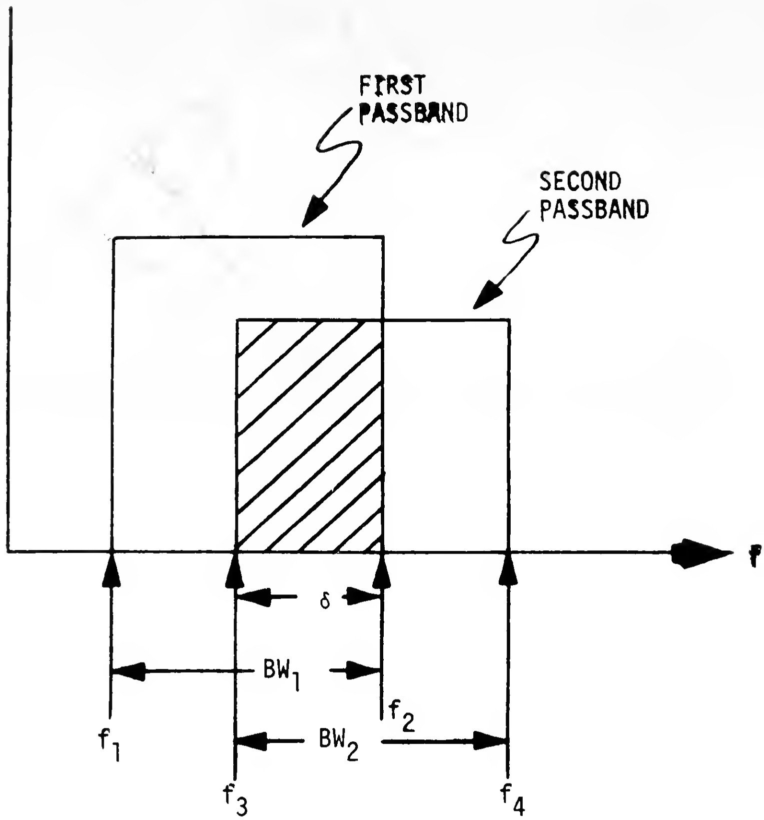
$$\theta = \omega T$$

$$\Delta\omega = \frac{\theta_2 - \theta_1}{T}$$

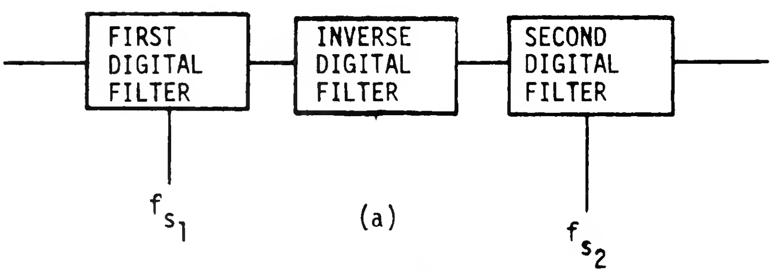
$$R_o = .90, .90, .90$$

$$R_p = .99, .97, .99$$

Fig. 7. A General Pole-Zero Plot for a Digital Passband Filter.



(b)



(a)

Fig. 8. Cascaded Digital Filters
With anConstant Bandwidth.

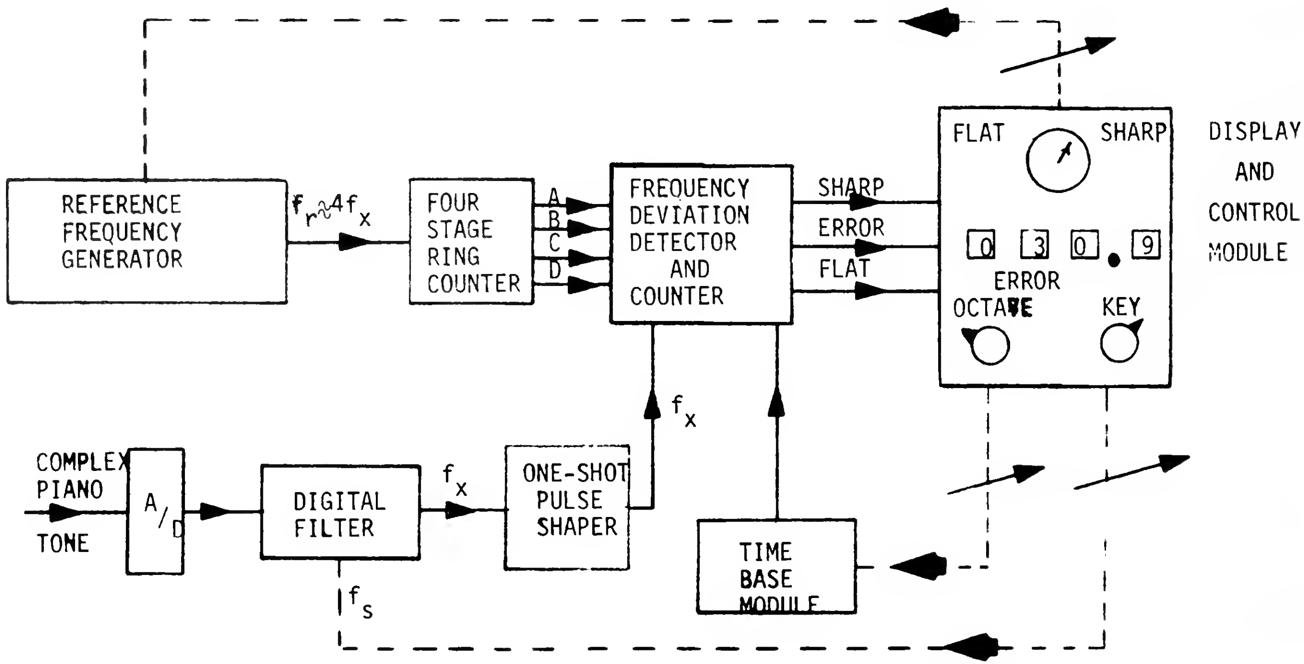


Fig. 9. General Block Diagram of Proposed Digital Piano Tuner.

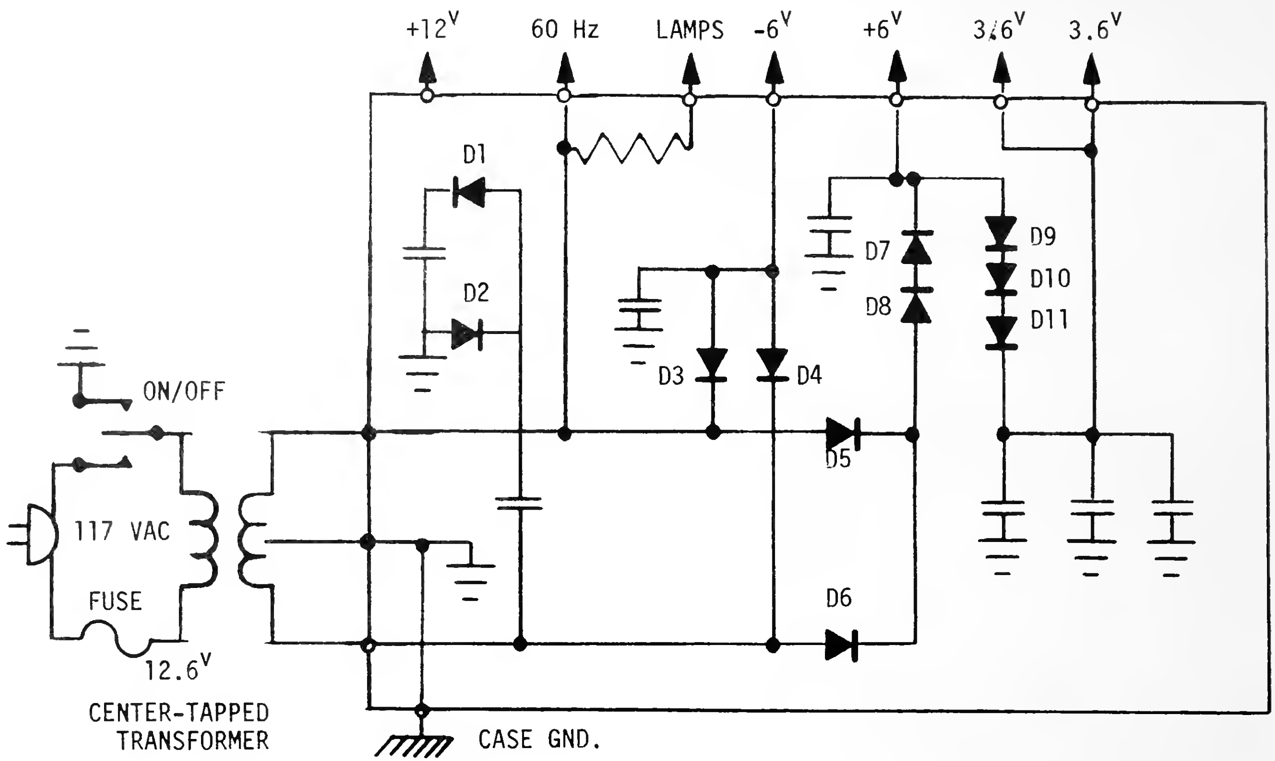


Fig. 10. Schematic of Proposed Power Supply.

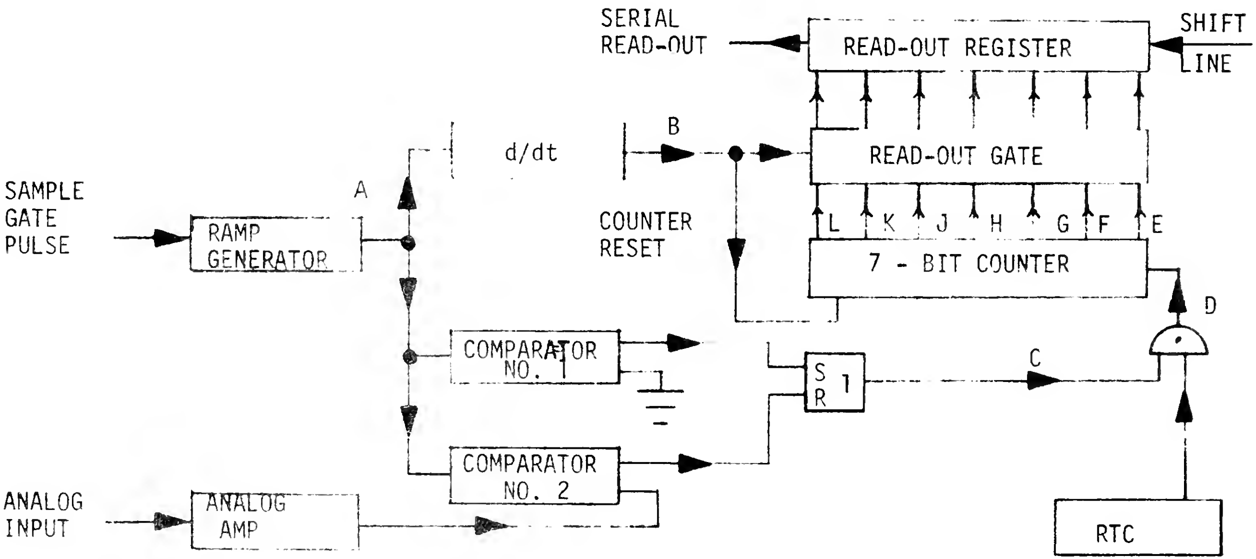


Fig. 11. Ramp A/D Converter.

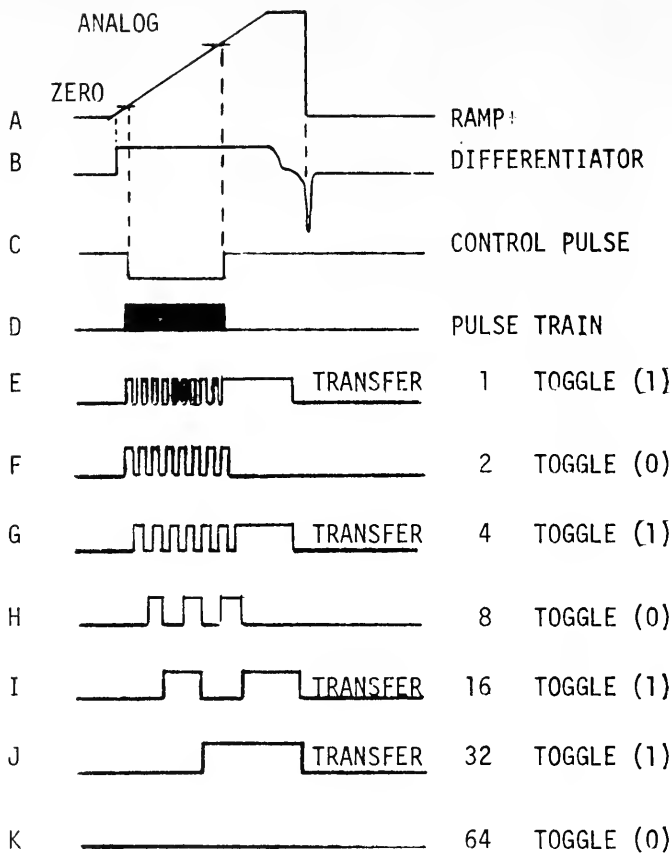
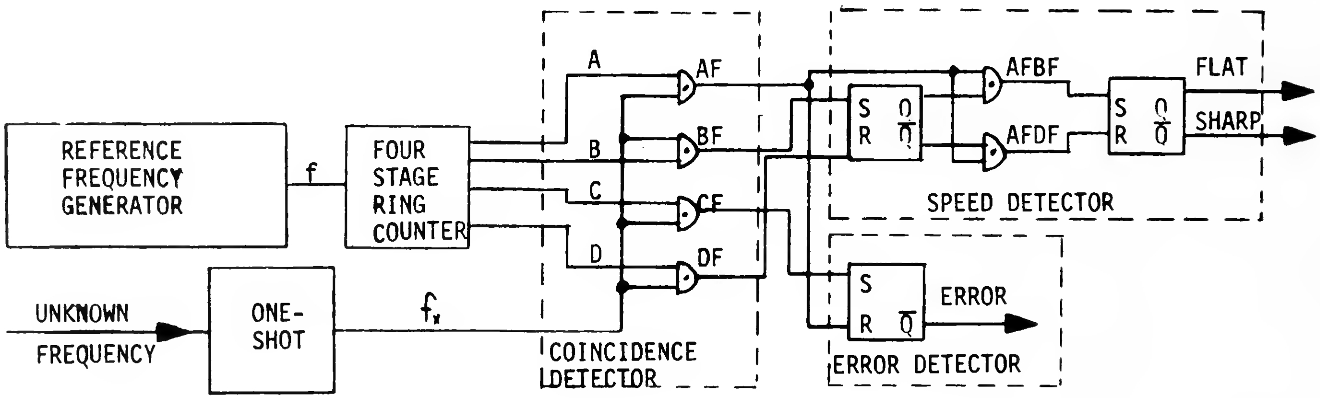


Fig. 12. Ramp A/D Converter Waveforms.



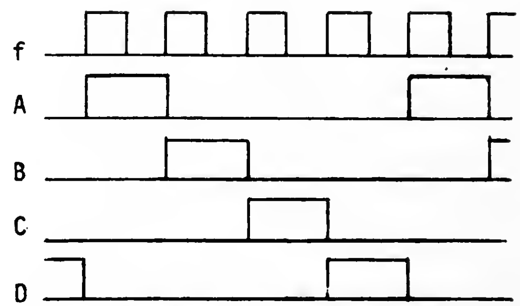


Fig. 13. Frequency Deviation Detector and Counter Module

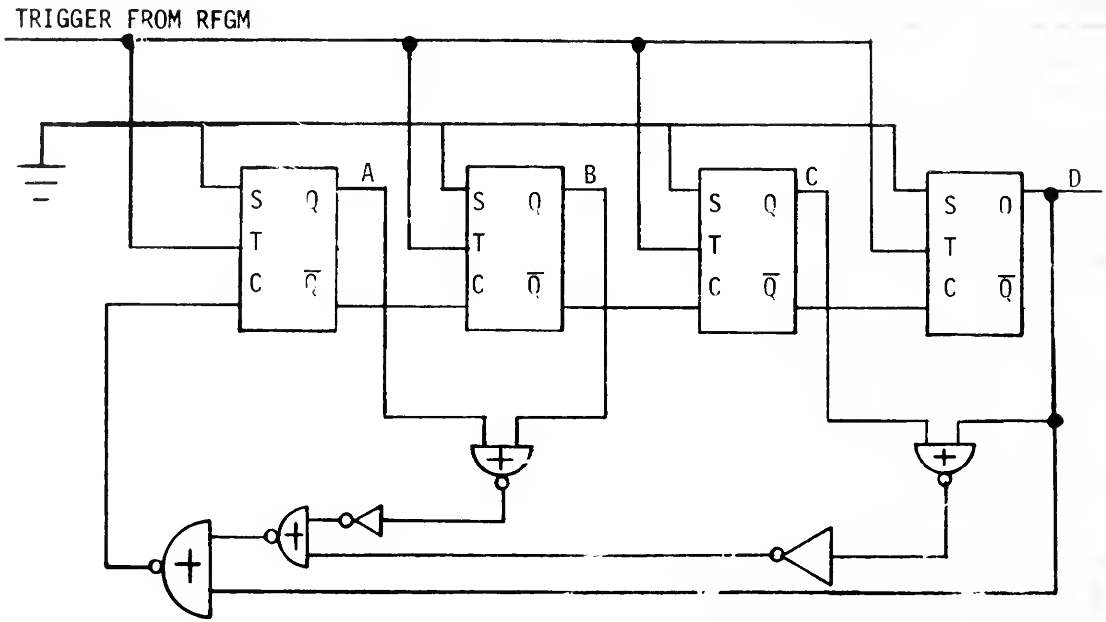


Fig. 14. Logic Diagram of the Four Stage Ring Counter

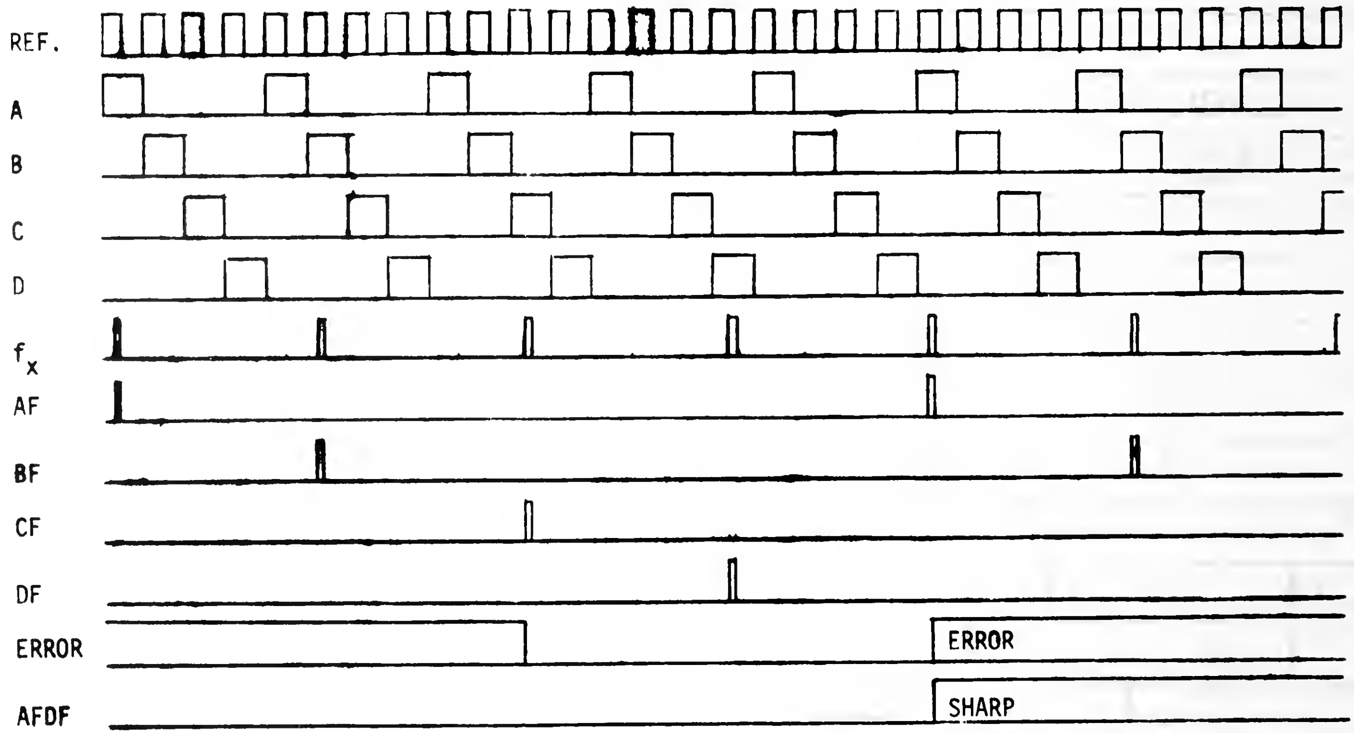


Fig. 15. Waveforms of the FDDCM When the Unknown Input is Sharp.

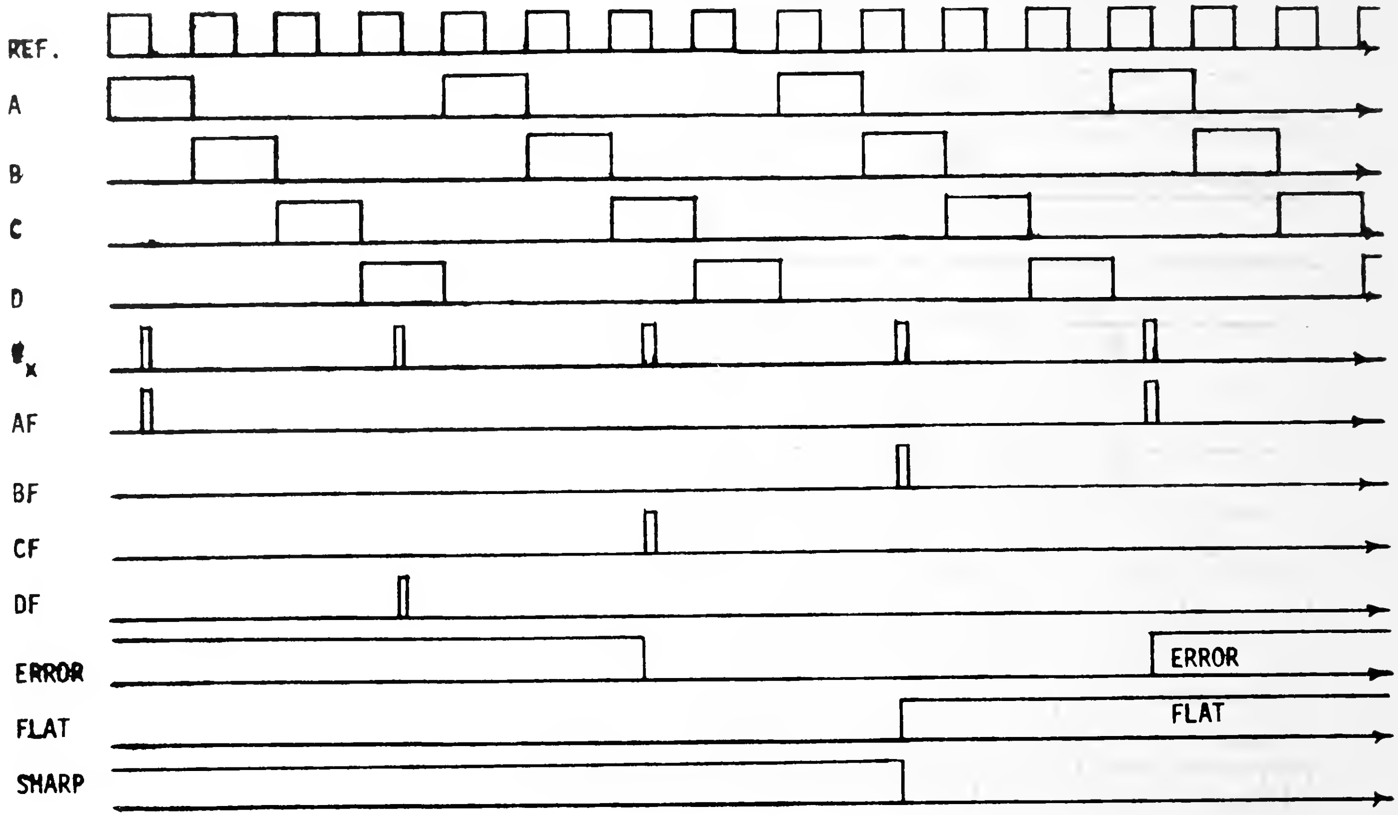


Fig. 16. Waveforms of the FDDCM When the Unknown Input is Flat.

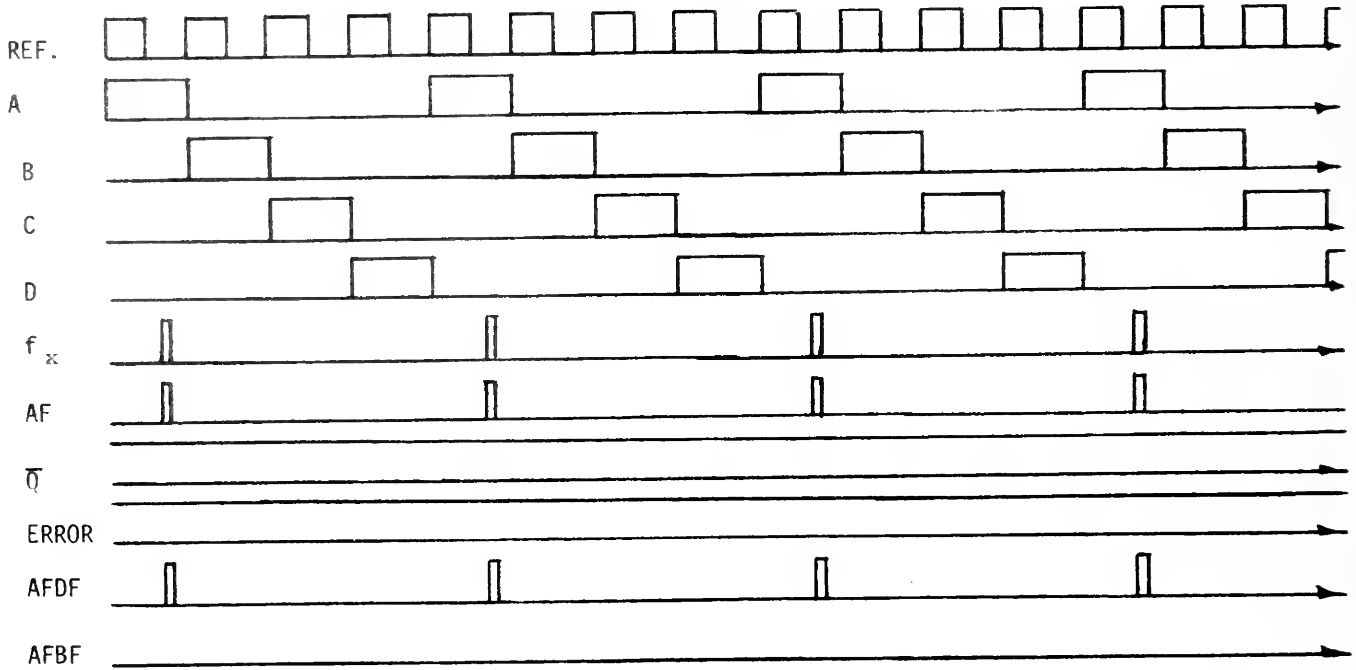


Fig. 17. Waveforms of the FDDCM When the Unknown is in Perfect Tune.

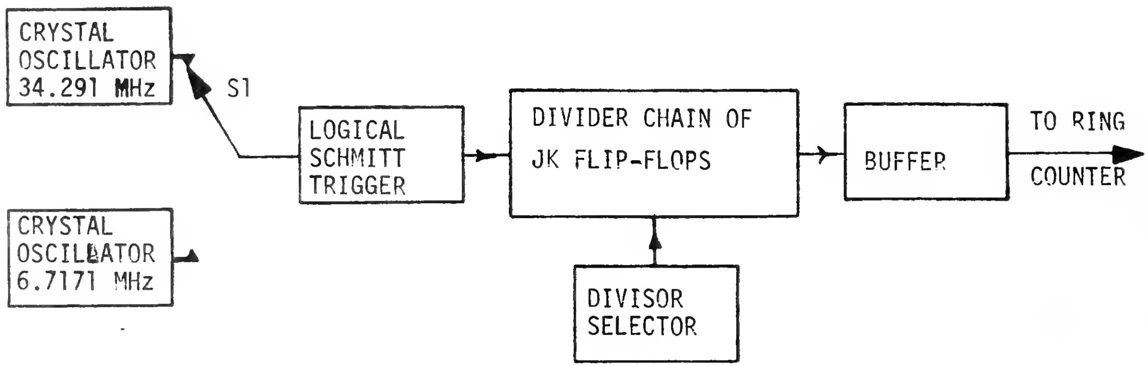


Fig. 18. Block Diagram of the Reference Frequency Generator Module.

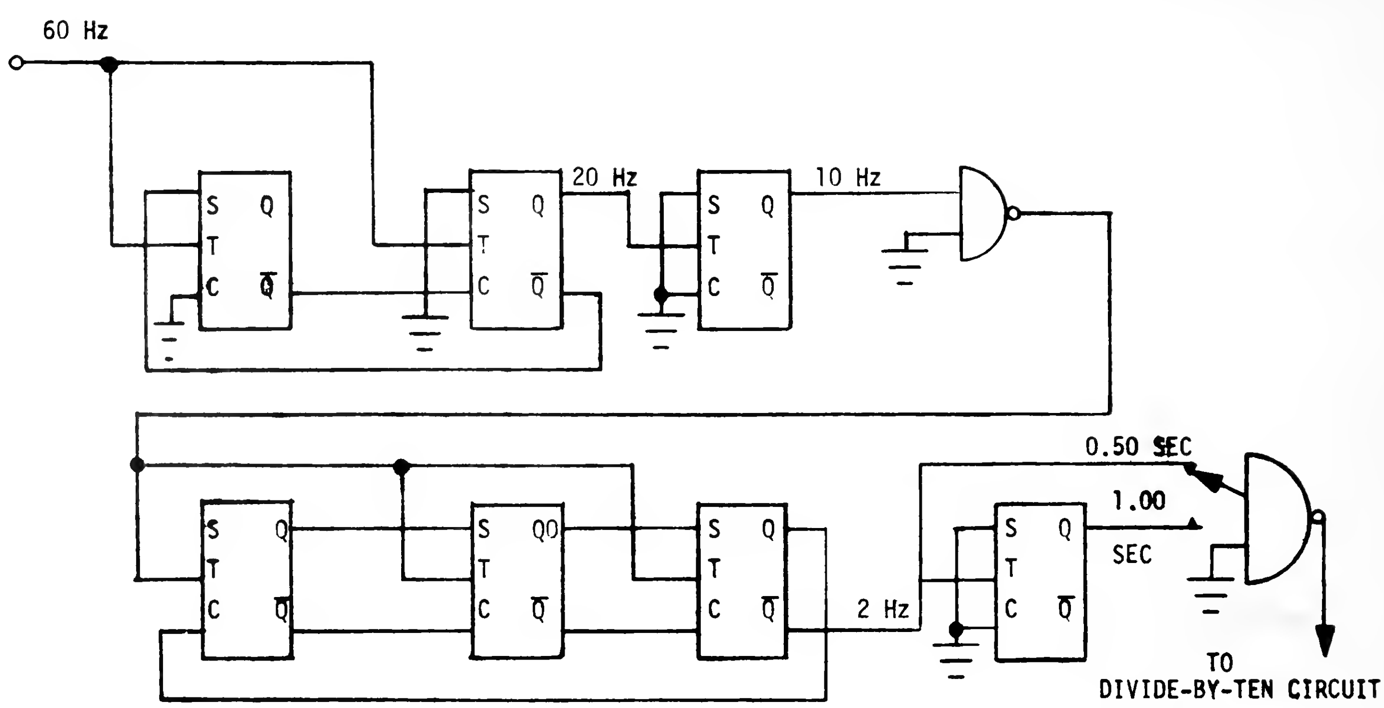


Fig. 19. Block Diagram of the Time Base Generator Module.

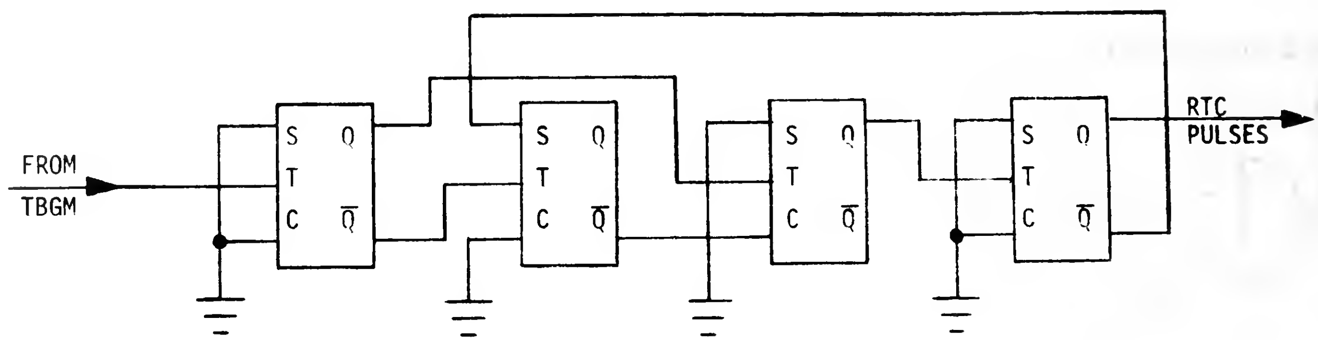


Fig. 20. Divide-By-Ten Circuit.

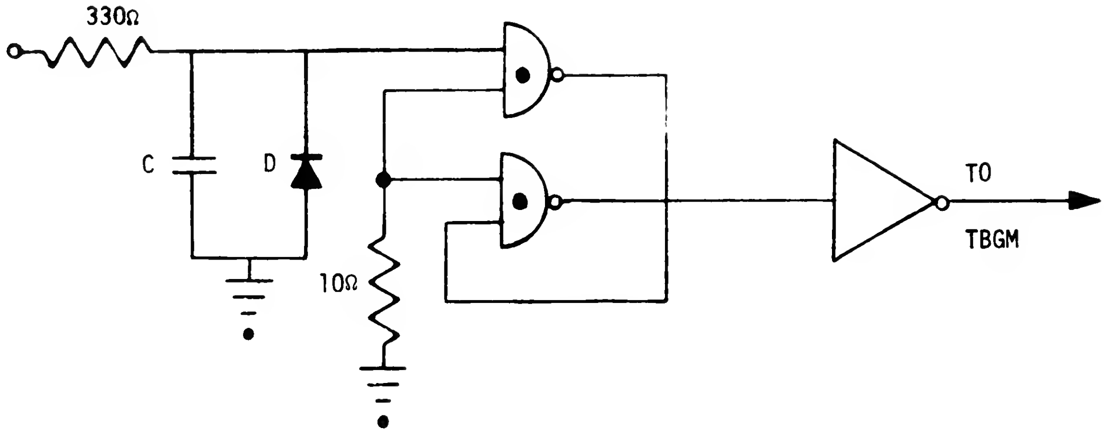


Fig. 21. Logical Schmitt Trigger.

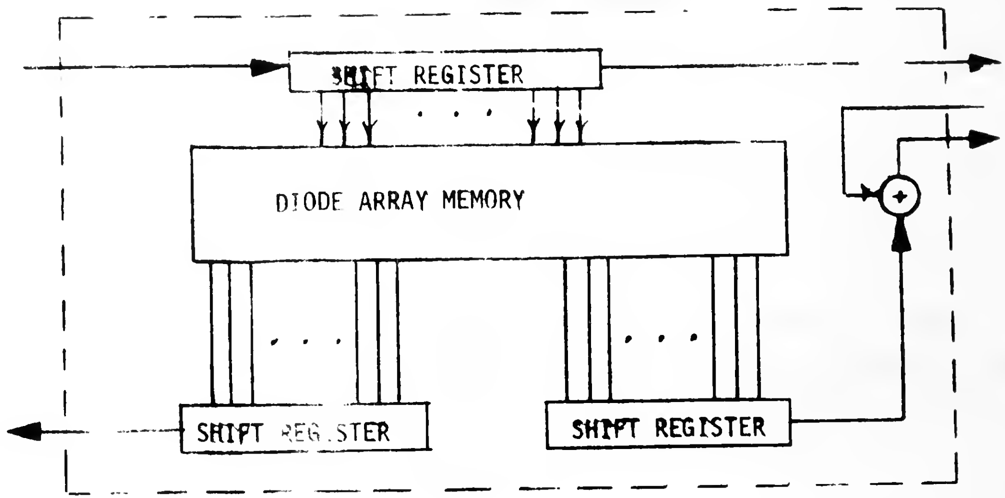
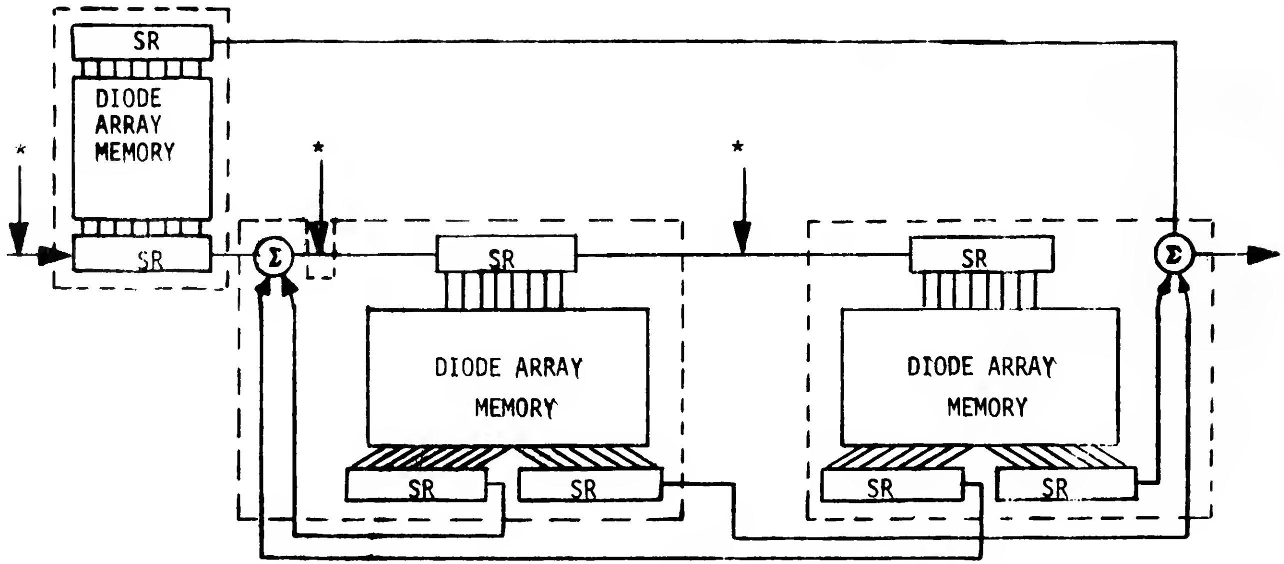


Fig. 22. Diode-Array Building Block for Fixed-Coefficient Digital Filter.



* BLOCKS FOR A HIGHER ORDER FILTER
MAY BE ADDED HERE

Fig. 23. Illustrating a Three Chip Second Order Recursive Serial Filter.

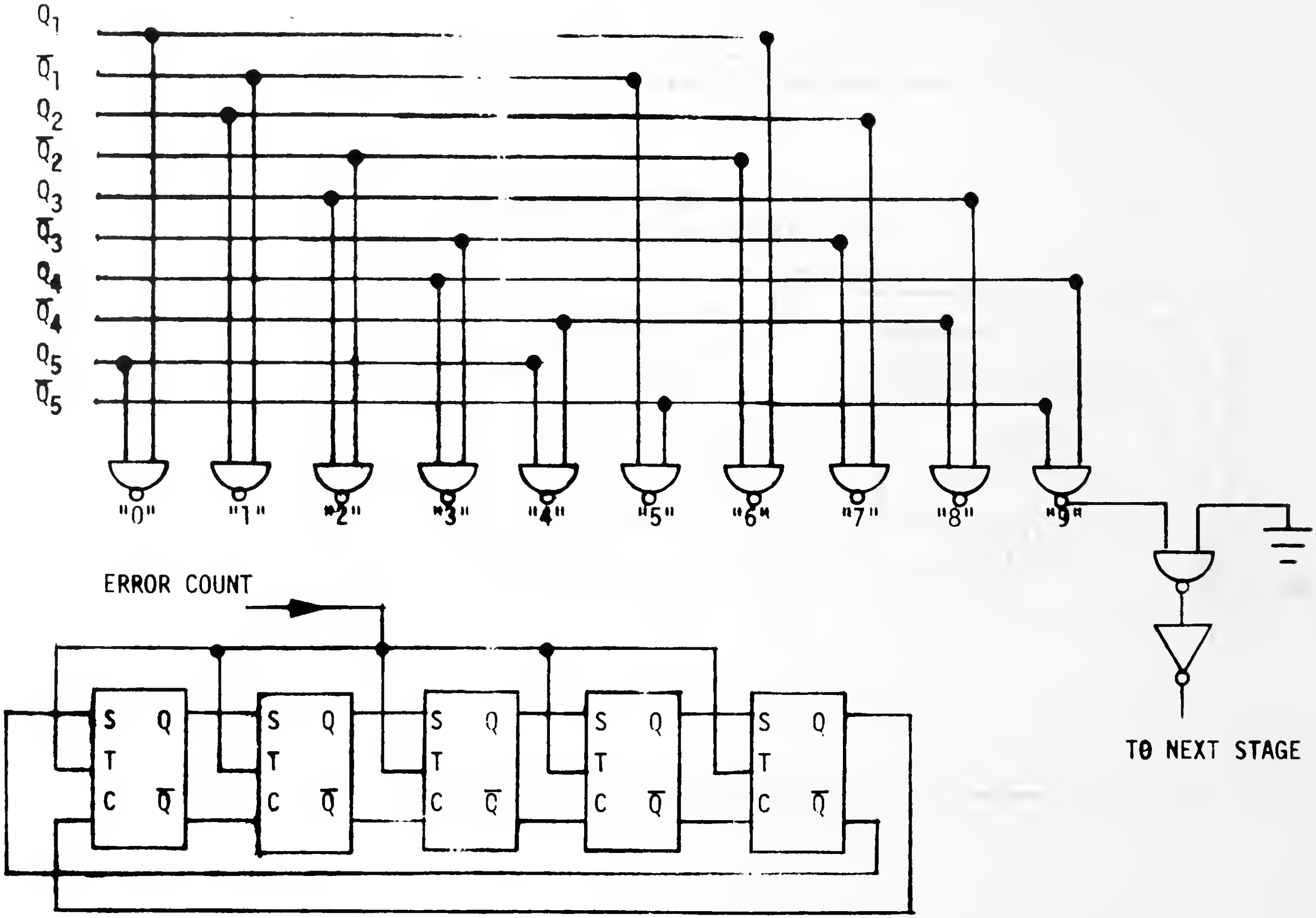


Fig. 24. Block Diagram and Decoding Scheme for a "10" Counter.

```
DESIGN OF A TUNEABLE DIGITAL FILTER BY M.W. HAGEE TOWARDS COMPLETION OF A THESIS FOR AN MS

THE FOLLOWING POLES AND ZEROES WERE READ INTO INBUF:

      NUMBER                POLE(RP)                         ZERO(RP)

        1             0.84411   0.51727J             0.76738   0.47025J
        2             0.84411  -0.51727J             0.76738  -0.47025J
        3             0.83363   0.49595J             0.77347   0.46016J
        4             0.83363  -0.49595J             0.77347  -0.46016J
        5             0.82273   0.47500J             0.77943   0.45000J
        6             0.82273  -0.47500J             0.77943  -0.45000J
        7             0.84632   0.47396J             0.78525   0.43976J
        8             0.84632  -0.47396J             0.78525  -0.43976J
        9             0.87003   0.47239J             0.79094   0.42944J
       10             0.87003  -0.47239J             0.79094  -0.42944J

THE FOLLOWING ARE THE PROPER FILTER COEFFICIENTS:

  NUMERATOR                       DENOMINATOR

 -0.7792919E 01 Z( 1)             -0.8433652E 01 Z( 1)
  0.2834116E 02 Z( 2)              0.3319263E 02 Z( 2)
 -0.6310678E 02 Z( 3)             -0.7998312E 02 Z( 3)
  0.9508841E 02 Z( 4)              0.1304175E 03 Z( 4)
 -0.1012020E 03 Z( 5)             -0.1502001E 03 Z( 5)
  0.7702196E 02 Z( 6)              0.1236957E 03 Z( 6)
 -0.4140474E 02 Z( 7)             -0.7195076E 02 Z( 7)
  0.1506186E 02 Z( 8)              0.2832016E 02 Z( 8)
 -0.3354658E 01 Z( 9)             -0.6924733E 01 Z( 9)
  0.3486865E 00 Z(10)              0.7675115E 00 Z(10)
```

THE FOLLOWING TABLE WAS GENERATED USING THETA= 30.0 AND DELTA THETA= 9.0 DEGREES.

KEY NO.	FUNDAMENTAL FREQUENCY	PARTIAL TO BE TUNED	PARTIAL FREQUENCY	LOWER FREQUENCY	UPPER FREQUENCY	LARGEST BANDWIDTH	SAMPLING FREQUENCY	RESULTANT BANDWIDTH
				0 OCTAVE				
1	27.5000	4.0	137.5000	110.0000	165.000	55.0000	1650.000	41.2500
2	29.1350	4.0	145.6750	116.5400	174.810	58.2700	1748.100	43.7025
3	30.8680	4.0	154.3400	123.4720	185.208	61.7360	1852.080	46.3020
				1 OCTAVE				
4	32.7030	4.0	163.5149	130.8120	196.218	65.4060	1962.179	49.0545
5	34.6480	4.0	173.2400	138.5920	207.888	69.2960	2078.880	51.9720
6	36.7080	4.0	183.5400	146.8320	220.248	73.4160	2202.479	55.0620
7	38.3910	4.0	191.9550	153.5640	230.346	76.7820	2303.458	57.5864
8	41.2030	4.0	206.0149	164.8120	247.218	82.4060	2472.179	61.8045
9	43.6540	4.0	218.2700	174.6160	261.924	87.3079	2619.237	65.4809
10	46.2490	4.0	231.2450	184.9960	277.494	92.4979	2774.937	69.3734
11	48.9990	4.0	244.9950	195.9960	293.994	97.9979	2939.937	73.4984
12	51.9130	4.0	259.5649	207.6519	311.478	103.8258	3114.779	77.8695
13	55.0000	4.0	275.0000	220.0000	330.000	110.0000	3300.000	82.5000
14	58.2700	4.0	291.3499	233.0799	349.620	116.5398	3496.198	87.4049
15	61.7350	4.0	308.6748	246.9398	370.410	123.4698	3704.096	92.6024
				2 OCTAVE				
16	65.4060	4.0	327.0298	261.6238	392.436	130.8118	3924.356	98.1089
17	69.2960	4.0	346.4797	277.1836	415.776	138.5920	4157.754	103.9438
18	73.4160	4.0	367.0798	293.6638	440.496	146.8318	4404.953	110.1238
19	77.7820	4.0	388.9099	311.1277	466.692	155.5642	4666.918	116.6729
20	82.4070	4.0	412.0349	329.6277	494.442	164.8142	4944.418	123.6104
21	87.3070	4.0	436.5349	349.2278	523.842	174.6140	5238.418	130.9604
22	92.4990	4.0	462.4949	369.9958	554.994	184.9978	5549.937	138.7484
23	97.9990	4.0	489.9949	391.9958	587.994	195.9978	5879.937	146.9984
24	103.8260	4.0	519.1299	415.3037	622.956	207.6521	6229.555	155.7389
25	110.0000	4.0	550.0000	440.0000	660.000	220.0000	6600.000	165.0000
26	116.5410	4.0	582.7048	466.1638	699.246	233.0818	6992.453	174.8113
27	123.4710	4.0	617.3547	493.8835	740.826	246.9421	7408.254	185.2062
				3 OCTAVE				
28	130.8130	4.0	654.0649	523.2520	784.878	261.6260	7848.777	196.2193
29	138.5910	4.0	692.9548	554.3638	831.546	277.1819	8315.453	207.8863
30	146.8320	4.0	734.1599	587.3279	880.992	293.6638	8809.918	220.2479
31	155.5630	4.0	777.8149	622.2520	933.378	311.1260	9333.777	233.3443
32	164.9140	4.0	824.0698	659.2556	988.884	329.6282	9888.836	247.2208
33	174.6140	4.0	873.0698	698.4558	1047.684	349.2278	10476.836	261.9207
34	184.9970	4.0	924.9849	739.9878	1109.982	369.9939	11099.816	277.4951
35	195.9980	4.0	979.9897	783.9917	1175.988	391.9958	11759.875	293.9968
36	207.6520	4.0	1038.2598	830.6077	1245.912	415.3040	12459.113	311.4775
37	220.0000	4.0	1100.0000	880.0000	1320.000	440.0000	13200.000	330.0000
38	233.0820	4.0	1165.4099	932.3279	1398.492	466.1638	13984.918	349.6228
39	246.9420	4.0	1234.7097	987.7676	1481.652	493.8840	14816.512	370.4126
				4 OCTAVE				
40	261.6260	4.0	1308.1299	1046.5039	1569.756	523.2520	15697.555	392.4385
41	277.1829	4.0	1385.9143	1108.7314	1663.097	554.3657	16630.969	415.7739
42	293.6648	4.0	1468.3240	1174.6592	1761.989	587.3296	17619.887	440.4968
43	311.1270	4.0	1555.6348	1244.5078	1866.762	622.2539	18667.613	466.6902
44	329.6279	4.0	1648.1396	1318.5117	1977.768	659.2559	19777.672	494.4417
45	349.2278	4.0	1746.1389	1396.9111	2095.367	698.4556	20953.664	523.8413
46	369.9939	4.0	1849.9695	1479.9756	2219.963	739.9878	22199.633	554.9907
47	391.9949	4.0	1959.9744	1567.9795	2351.969	783.9897	23519.691	587.9922
48	415.3049	4.0	2076.5247	1661.2197	2491.830	830.6099	24918.293	622.9573
49	440.0000	4.0	2200.0000	1760.0000	2640.000	880.0000	26400.000	660.0000
50	466.1638	4.0	2330.8191	1864.6553	2796.983	932.3276	27969.824	699.2454
51	493.8828	4.0	2469.4141	1975.5312	2963.297	987.7656	29632.969	740.8240
				5 OCTAVE				
52	523.2510	3.0	2093.0039	1569.7529	2616.255	1046.5020	25116.043	627.9009
53	554.3650	3.0	2217.4600	1663.0950	2771.825	1108.7300	26609.516	665.2378
54	587.3298	3.0	2349.3193	1761.9895	2936.649	1174.6597	29191.828	704.7957
55	622.2539	3.0	2489.0156	1866.7617	3111.270	1244.5078	29868.187	746.7046
56	659.2549	3.0	2637.0195	1977.7646	3296.274	1318.5098	31644.230	791.1057
57	698.4558	3.0	2793.8232	2095.3674	3492.279	1396.9116	33525.875	838.1457
58	739.9888	3.0	2959.9551	2219.9663	3699.944	1479.9775	35519.430	887.9854
59	783.9910	3.0	3135.9639	2351.9729	3919.955	1567.9819	37631.531	940.7881
60	830.6089	3.0	3322.4355	2491.8267	4153.043	1661.2163	39869.199	996.7297
61	880.0000	3.0	3520.0000	2640.0000	4400.000	1760.0000	42240.000	1056.0000
62	932.3279	3.0	3729.3115	2796.9836	4661.637	1864.6531	44751.730	1118.7932
63	987.7668	3.0	3951.0674	2963.3005	4938.832	1975.5315	47412.797	1185.3196
				6 OCTAVE				
64	1046.5020	1.0	2093.0039	1046.5020	3139.506	2093.0039	25116.043	627.9009
65	1108.7310	1.0	2217.4619	1108.7310	3326.193	2217.4619	26609.539	665.2383
66	1174.6589	1.0	2349.3179	1174.6589	3523.977	2349.3179	28191.812	704.7952
67	1244.5078	1.0	2489.0156	1244.5078	3733.523	2489.0156	29868.187	746.7046
68	1318.5098	1.0	2637.0195	1318.5098	3955.529	2637.0195	31644.230	791.1057
69	1396.9128	1.0	2793.8257	1396.9128	4190.738	2793.8254	33525.906	838.1475
70	1479.9778	1.0	2959.9556	1479.9778	4439.930	2959.9519	35519.465	887.9863
71	1567.9819	1.0	3135.9639	1567.9819	4703.945	3135.9634	37631.531	940.7881
72	1661.2190	1.0	3322.4380	1661.2190	4983.656	3322.4373	39869.230	996.7307
73	1760.0000	1.0	3520.0000	1760.0000	5280.000	3520.0000	42240.000	1056.0000
74	1864.6548	1.0	3729.3096	1864.6548	5593.961	3729.3062	44751.699	1118.7922
75	1975.5330	1.0	3951.0659	1975.5330	5926.598	3951.0647	47412.766	1185.3191
				7 OCTAVE				
76	2093.0049	0.0	2093.0049	0.0	4186.008	4186.0078	25116.055	627.9011
77	2217.4609	0.0	2217.4609	0.0	4434.922	4434.9219	26609.531	665.2380
78	2349.3179	0.0	2349.3179	0.0	4698.633	4698.6328	28191.812	704.7952
79	2489.0159	0.0	2489.0159	0.0	4978.031	4978.0312	29868.187	746.7046
80	2637.0210	0.0	2637.0210	0.0	5274.039	5274.0391	31644.250	791.1062
81	2793.8259	0.0	2793.8259	0.0	5587.648	5587.6484	33525.910	638.1477
82	2959.9548	0.0	2959.9548	0.0	5919.906	5919.9062	35519.430	887.9854
83	3135.9639	0.0	3135.9639	0.0	6271.926	6271.9258	37631.531	940.7881
84	3322.4380	0.0	3322.4380	0.0	6644.875	6644.8750	39869.230	996.7307
85	3520.0000	0.0	3520.0000	0.0	7040.000	7040.0000	42240.000	1056.0000
86	3729.3098	0.0	3729.3098	0.0	7458.617	7458.6172	44751.699	1118.7922
87	3951.0659	0.0	3951.0659	0.0	7902.129	7902.1289	47412.766	1185.3191
88	4186.0078	0.0	4186.0078	0.0	8372.016	8372.0156	50232.066	1255.8015

```
C
C
C
C
C PROGRAM TO COMPUTE THE FILTER COEFFICIENTS OF A DIGITAL FILTER AND TO
C PLOT THE MAGNITUDE OF THE DESIGNED FILTER AS A FUNCTION OF FREQUENCY.
C THE COMPLEX POLES AND ZEROES ARE THE ONLY VALUES REQUIRED TO BE READ
C INTO THE INBUF
C
C
C
C
C
C LABEL GRAPHS
C
      REAL*8ITITLE(12)/96HMAGNITUDE OF TRANSFER FUNCTION VS FREQUENCY
     1  DESIGN OF A DIGITAL FILTER -- M.W. HAGEE DECK 1    /
      REAL*4LABEL1/'FONE'/,LABEL2/'FTWO'/,LABEL3/'THRE'/,LABEL4/'FOUR'/,
     1LABEL5/'FIVE'/
C
C COMPLEX QUANTITIES
C
      COMPLEX*16 ZERO(10),POLE(10),C1(10),C2(10)
      COMPLEX*8 HN(10),HD(10),Y1,Y2,Y3,SCALE1,SCALE2
C
C REAL QUANTITIES
C
      REAL*8 STEP1,STEP2
      REAL*4 NC(20),DC(20),YSUM(900),SINE,CSINE,RPZ,RPP,F(900),DB(900),G
     1,ST,SF,OMEGA,DELTA
C
C N IS THE NUMBER OF COMPLEX POLES OR ZEROES IN THE Z PLANE
C DELTA IS FREQUENCY INCREMENT IN RADIANS
C NPT IS THE NUMBER OF POINTS TO BE PLOTTED
C
      N=10
      NPT=900
      PI=3.1415927
      DELTA=2.*PI*1.
C
C SAMPLING INTERVAL
C
      ST=1.0/2750.
C
C ADJUST GAIN OF BANDPASS
C
      SCALE1=CMPLX(1.0,0.0)
      SCALE2=CMPLX(1.0,0.0)
```

```
      WRITE(6,210)
  210 FORMAT(//,' DESIGN OF A TUNEABLE DIGITAL FILTER BY M.W. HAGEE TOWA
     1RDS COMPLETION OF A THESIS FOR AN MS.',///,' THE FOLLOWING POLES A
     2ND ZEROES WERE READ INTO INBUF:',//,10X,'NUMBER',20X,'POLE(RP)',20
     3X,'ZERO(RP)',/)
C
C READ IN REAL VALUES OF POLES AND ZEROES
C
      DO 100 I=1,N
      READ(5,200) RPP,RPZ,CSINE,SINE
  200 FORMAT(4F10.6)
      ZERO(I)=CMPLX(RPZ*CSINE,RPZ*SINE)
      POLE(I)=CMPLX(RPP*CSINE,RPP*SINE)
      WRITE(6,204) I,POLE(I),ZERO(I)
  204 FORMAT(12X,I2,16X,F8.5,2X,F8.5,'J',15X,F8.5,2X,F8.5,'J')
  100 CONTINUE
C
C COMPUTE ALL COEFFICIENTS OF FILTER BOTH DENOMINATOR AND NUMERATOR
C
      CALL FINCOF(N,POLE,C2)
      CALL FINCOF(N,ZERO,C1)
      K=N+1
      DO 45 I=1,N
      K=K-1
      STEP1=CDABS(C1(K))
      STEP2=CDABS(C2(K))
      NC(I)=((-1.0)**I)*SNGL(STEP1)
      DC(I)=((-1.0)**I)*SNGL(STEP2)
   45 CONTINUE
      WRITE(6,201)
  201 FORMAT(//,' THE FOLLOWING ARE THE PROPER FILTER COEFFICIENTS:',//)
      WRITE(6,203)
  203 FORMAT('   NUMERATOR                          DENOMINATOR'/)
      DO 104 I=1,N
      WRITE(6,202)NC(I),I,DC(I),I
  202 FORMAT(2X,E14.7,' Z(', I2, ')', 10X, E14.7, ' Z(', I2, ')')
  104 CONTINUE
      TALLY=0.0
      CHECK=0.0
      DO 1 I=1,N
      CHECK=CHECK+NC(I)
      TALLY=TALLY+DC(I)
    1 CONTINUE
      ZMAG=CHECK/TALLY
      WRITE(6,2)CHECK,TALLY,ZMAG
    2 FORMAT(' AT FREQ=0 CHECK= ',E14.7,' TALLY= ',E14.7,' ZMAG= ',E14.7
     1,//)
C
```

```
C COMPUTE THE MAGNITUDE OF THE TRANSFER FUNCTION AS A FUNCTION OF
C FREQUENCY FOR VARIOUS SAMPLING VALUES
C
      LL=1
      DO 105 L=1,5
C
C STARTING FREQUENCY IN RADIANS
C
      OMEGA=2.*PI*100.
      WRITE(6,211)
  211 FORMAT(///,' THE FOLLOWING TABLE COMPARES DB OF FILTER TRANSFER FU
     1NCTION WITH FREQUENCY:',//,' STEP',10X,'FREQ',10X'MAG(DB)',/)
      SF=1.0/ST
      WRITE(6,215) SF
  215 FORMAT(' SAMPLING FREQUENCY = ',E12.6,' HERZ FOR THIS TABLE.',/)
      DO 106 J=1,NPT
      G=0.0
      DO 107 I=1,N
      G=G+1.0
      HN(I)=CMPLX(NC(I)*COS(OMEGA*G*ST),-NC(I)*SIN(OMEGA*G*ST))
      HD(I)=CMPLX(DC(I)*COS(OMEGA*G*ST),-DC(I)*SIN(OMEGA*G*ST))
  107 CONTINUE
      Y1=SCALE1+HN(1)+HN(2)
      Y2=SCALE2+HD(1)+HD(2)
      DO 108 I=3,M,2
      K=I+1
      Y1=Y1+HN(I)+HN(K)
      Y2=Y2+HD(I)+HD(K)
  108 CONTINUE
      YSUM(J)=CABS(Y1/Y2)
C
C COMPUTE MAGNITUDE OF FILTER TRANSFER FUNCTION IN DB
C
      DB(J)=20.*ALOG10(YSUM(J))
      F(J)=CMEGA/(2.*PI)
      WRITE(6,212) J,F(J),DB(J)
  212 FORMAT(2X,I3,10X,F6.1,7X,F10.5)
C
C INCREMENT THE STARTING FREQUENCY BY DELTA
C
      OMEGA=OMEGA+DELTA
  106 CONTINUE
C
C PLOT MAGNITUDE(DB) AS A FUNCTION OF FREQUENCY
C
      IF(LL.EQ.1) GO TO 30
      IF(LL.EQ.2) GO TO 31
      IF(LL.EQ.3) GO TO 32
```

```
      IF(LL.EQ.4) GO TO 33
      IF(LL.EQ.5) GO TO 34
   30 CALL DRAW(NPT,F,DB,0,0,LABEL1,ITITLE,0.,0.,0,0,0,0,8,8,1,LAST)
      GO TO 800
   31 CALL DRAW(NPT,F,DB,0,0,LABEL2,ITITLE,0.,0.,0,0,0,0,8,8,1,LAST)
      GO TO 800
   32 CALL DRAW(NPT,F,DB,0,0,LABEL3,ITITLE,0.,0.,0,0,0,0,8,8,1,LAST)
      GO TO 800
   33 CALL DRAW(NPT,F,DB,0,0,LABEL4,ITITLE,0.,0.,0,0,0,C,8,8,1,LAST)
      GO TO 800
   34 CALL DRAW(NPT,F,DB,0,0,LABEL5,ITITLE,0.,0.,0,0,0,C,8,8,1,LAST)
  800 LL=LL+1
      ST=ST/1.5
  105 CONTINUE
      END


      SUBROUTINE FINCOF(N,R,C)
      COMPLEX*16 R(N),C(N)
      IF(N.LE.0) RETURN
      DO 10 I=1,N
   10 C(I)=R(I)
      K=N
      M=N-1
      DO 20 L=1,M
      DO 30 I=2,K
   30 C(I)=C(I)+C(I-1)
      K=K-1
      DO 20 I=1,K
      J=I+L
      C(I)=R(J)*C(I)
   20 CONTINUE
      K=N/2
      K=2*K/(N-K)
      DO 40 I=K,N,2
   40 C(I)=-C(I)
      RETURN
      END
0.99          0.90     0.85264    0.52250
0.99          0.90     0.85264   -0.52250
0.97          0.90     0.85941    0.51129
0.97          0.90     0.85941   -0.51129
0.95          0.90     0.86603    0.50000
0.95          0.90     0.86603   -0.50000
0.97          0.90     0.87250    0.48862
0.97          0.90     0.87250   -0.48862
```

0.99	0.90	0.87882	0.47716
0.99	0.90	0.87882	-0.47716

```
C
C
C
C     PROGRAM TO DETERMINE THE SAMPLING FREQUENCY AND RESULTANT BANDWIDTH
C     FOR A TUNEABLE DIGITAL FILTER.
C     PAR IS THE PARTIAL OF THE FUNDAMENTAL TO BE FILTERED.
C     THETA IS THE CENTER ANGLE OF BANDWIDTH IN THE Z-DOMAIN.
C     "F" IS THE FUNDAMENTAL FREQUENCY, "FC" IS THE PARTIAL FREQUENCY, "FS"
C     IS THE REQUIRED SAMPLING FREQUENCY, "DELF" IS THE BANDWIDTH OF THE
C     FILTER AT THE SPECIFIC SAMPLING FREQUENCY "FS".
C     FLOW IS THE FIRST PARTIAL FREQUENCY BELOW THE ONE DESIRED.  FHIGH IS
C     THE NEXT HIGHEST PARTIAL FREQUENCY ABOVE THE DESIRED ONE.  FMAX IS THE
C     LARGEST BANDWIDTH ALLOWABLE IN ORDER TO BE ABLE TO TUNE THE DESIRED
C     PARTIAL FROM THE COMPLEX SOUND.
C
C
C
C
C
C
C
C     DIMENSION TO HANCLE ALL 88 PIANO KEYS.
C
C
      DIMENSION F(88),FC(88),FS(88),DELF(88),DELSF(87),DELBW(87)
      DIMENSION FLOW(88),FHIGH(88),FMAX(88)
C
C
C     SET INTIAL VALUES FOR ALL VARIABLES.
C
C
      PAR=5.0
      THETA=30.0
      DELTH=9.0
C
C     READ IN THE 88 FUNDAMENTAL FREQUENCIES OF THE EQUAL TEMPERED PIANO.
C
      DO 100 I=1,88
      READ(5,10) F(I)
   10 FORMAT(F10.3)
  100 CONTINUE
C
C     COMPUTE ALL DESIRED QUANTITIES.
C
      DO 101 I=1,88
C
C     REQUIRED PARTIAL.
C
```

```
      FC(I)=PAR*F(I)
C
C NEXT LOWEST PARTIAL
C
      FLOW(I)=FC(I)-F(I)
C
C NEXT HIGHEST PARTIAL ABCVE THE DESIRED ONE.
C
      FHIGH(I)=FC(I)+F(I)
C
C MAX ALLOWABLE BANDWIDTH.
C
      FMAX(I)=FHIGH(I)-FLOW(I)
C
C SAMPLING FREQUENCY.
C
      FS(I)=FC(I)*360.0/THETA
C
C BANDWIDTH.
C
      DELF(I)=DELTH*FS(I)/360.
C
C DECREASE THE PARTIAL NUMBER DEPENDING UPON THE REGISTER NUMBER.
C
      IF(I.EQ.51.CR.I.EQ.75) PAR=PAR-1.0
      IF(I.EQ.63) PAR=PAR-2.0
  101 CONTINUE
C
C OUTPUT ALL PERTINENT DATA.
C
      WRITE(6,200) THETA, DELTH
  200 FORMAT(///,' THE FOLLOWING TABLE WAS GENERATED USING THETA= ',F5.1
     1,' AND DELTA THETA= ',F5.1,' DEGREES.',//)
      WRITE(6,301)
      WRITE(6,302)
  301 FORMAT(' KEY  FUNDAMENTAL  PARTIAL TO   PARTIAL     LOWER',6X,'UPP
     1ER',5X,'LARGEST   SAMPLING   RESULTANT')
  302 FORMAT(' NO.    FREQUENCY    BE TUNED   FREQUENCY  FREQUENCY  FREQU
     1ENCY  BANDWIDTH  FREQUENCY  BANDWIDTH',///)
      PAR=4.0
      IOCT=-1
      DO 102 I=1,88
      IF(I.NE.4.AND.I.NE.16.AND.I.NE.28.AND.I.NE.40.AND.I.NE.52.AND.I.NE
     1.64.AND.I.NE.76.AND.I.NE.1) GO TO 500
      IOCT=IOCT+1
      WRITE(6,300)IOCT
  300 FORMAT(/,44X,I1,' OCTAVE',/)
  500 CONTINUE
```

```
      WRITE(6,202)I,F(I),PAR,FC(I),FLOW(I),FHIGH(I),FMAX(I),FS(I),DELF(I
     1)
  202 FORMAT(1X,I2,4X,F9.4,6X,F3.1,6X,F9.4,2X,F9.4,2X,F9.3,2X,F9.4,2X,F9
     1.3,2X,F9.4)
      IF(I.EQ.51.OR.I.EQ.75) PAR=PAR-1.0
      IF(I.EQ.63) PAR=PAR-2.0
  102 CONTINUE
      WRITE(6,209)
      WRITE(6,210)
      WRITE(6,211)
  209 FORMAT(///,' THE FOLLOWING TABLE WAS GENERATED TO ILLUSTRATE THE R
     1EQUIRED CHANGE IN SAMPLING FREQUENCY AND')
  210 FORMAT(' THE RESULTANT CHANGE IN BANDWIDTH OF THE DIGITAL FILTER A
     1S EACH PROGRESSIVE KEY IS TUNED',//)
  211 FORMAT(' FROM KEY  TO  KEY     DELTA SAMPLING FREQ    DELTA BANDWI
     1DTH',/)
      I=0
      K=1
C
C COMPUTE THE CHANGE IN SAMPLING FREQUENCY AS A FUNCTION OF FREQUENCY
C
      DO 103 J=1,87
      I=I+1
      K=K+1
      DELSF(J)=FS(K)-FS(I)
      DELBW(J)=DELF(K)-DELF(I)
      WRITE(6,203)I,K,DELSF(J),DELBW(J)
  203 FORMAT(7X,I2,7X,I2,6X,F12.4,' HERZ',5X,F8.2,' HERZ')
  103 CONTINUE
C
C PLOT GRAPHS.
C
      WRITE(6,205)
  205 FORMAT(1H1,///,' SAMPLING FREQUENCY AS A FUNCTION OF PARTIAL FREQU
     1ENCY',///)
      CALL PLOTP(FC,FS,88,0)
      WRITE(6,206)
  206 FORMAT(1H1,///,' BANDWIDTH AS A FUNCTION OF SAMPLING FREQUENCY',//
     1/)
      CALL PLOTP(FS,DELF,88,0)
      WRITE(6,207)
  207 FORMAT(1H1,///,' BANDWIDTH AS A FUNCTION OF PARTIAL FREQUENCY',///
     1)
      CALL PLOTP(FC,DELF,88,0)
      WRITE(6,208)
  208 FORMAT(1H1,///,' CHANGE IN SAMPLING FREQUENCY AS A FUNCTION OF PAR
     1TIAL FREQUENCY',///)
      CALL PLOTP(FC,DELSF,87,0)
```

```
      END
C
C THE 88 FREQUENCIES FOR THE EQUAL-TEMPERED PIANO IN HERZ
C
27.500
29.135
30.868
32.703
34.648
36.708
38.391
41.203
43.654
46.249
48.999
51.913
55.000
58.270
61.735
65.406
69.296
73.416
77.782
82.407
87.307
92.499
97.999
103.826
110.000
116.541
123.471
130.813
138.591
146.832
155.563
164.814
174.614
184.997
195.998
207.652
220.000
233.082
246.942
261.626
277.183
293.665
311.127
329.628
```

349.228
369.994
391.995
415.305
440.000
466.164
493.883
523.251
554.365
587.330
622.254
659.255
698.456
739.989
783.991
830.609
880.000
932.328
987.767
1046.502
1108.731
1174.659
1244.508
1318.510
1396.913
1479.978
1567.982
1661.219
1760.000
1864.655
1975.533
2093.005
2217.461
2349.318
2489.016
2637.021
2793.826
2959.955
3135.964
3322.438
3520.000
3729.310
3951.066
4186.009

```
C
C
C
C
C     PROGRAM TO COMPUTE THE FREQUENCY RESPONSE OF A DIGITAL FILTER BY
C     IMPLEMENTING EQUATION (13).
C     THE COMPLEX POLES AND ZEROES ARE THE ONLY VALUES REQUIRED TO BE READ
C     INTO THE INBUF
C
C
C
C
C
C     LABEL GRAPHS
C
      REAL*8ITITLE(12)/96HMAGNITUDE OF TRANSFER FUNCTION VS FREQUENCY
     1  DESIGN OF A DIGITAL FILTER -- M.W. HAGEE DECK 1     /
      REAL*4LABEL1/'FONE'/,LABEL2/'FTWC'/,LABEL3/'THRE'/,LABEL4/'FOUR'/,
     1LABEL5/'FIVE'/
C
C     COMPLEX QUANTITIES
C
      COMPLEX*8 HN(20),HD(20),Y1,Y2,Y3,SCALE1,SCALE2,ZERO(20),POLE(20),P
     1(20),SUM,ZSUM(50),PSUM(50),ZEXP
C
C     REAL QUANTITIES
C
      REAL*4 NC(2C),DC(20),YSUM(900),SINE,CSINE,RPZ,RPP,F(900),DB(900),G
     1,ST,SF,OMEGA,DELTA
C
C     N IS THE NUMBER CF COMPLEX POLES OR ZEROES IN THE Z PLANE
C     DELTA IS FREQUENCY INCREMENT IN RADIANS
C     NPT IS THE NUMBER OF PCINTS TO BE PLOTTED
C
      N=10
      NPT=900
      PI=3.1415927
      DELTA=2.*PI*1.
C     SAMPLING INTERVAL
C
      ST=1.0/2750.
C
C     ADJUST GAIN OF BANDPASS
C
      SCALE1=CMPLX(1.0,0.0)
      SCALE2=CMPLX(1.0,0.0)
      WRITE(6,210)
  210 FORMAT(//,' DESIGN CF A TUNEABLE DIGITAL FILTER BY M.W. HAGEE TOWA
```

```
     1RDS COMPLETION OF A THESIS FOR AN MS.',///,' THE FOLLOWING POLES A
     2ND ZEROES WERE READ INTO INBUF:',//,10X,'NUMBER',20X,'POLE(RP)',20
     3X,'ZERO(RP)',/)
C
C READ IN REAL VALUES OF POLES AND ZEROES
C
      DO 100 I=1,N
      READ(5,200) RPP,RPZ,CSINE,SINE
  200 FORMAT(4F10.6)
      ZERO(I)=CMPLX(RPZ*CSINE,RPZ*SINE)
      POLE(I)=CMPLX(RPP*CSINE,RPP*SINE)
      WRITE(6,204) I,POLE(I),ZERO(I)
  204 FORMAT(12X,I2,16X,F8.5,2X,F8.5,'J',15X,F8.5,2X,F8.5,'J')
  100 CONTINUE
      LL=1
      DO 105 L=1,5
C
C STARTING FREQUENCY IN RADIANS
C
      OMEGA=0.0
      WRITE(6,211)
  211 FORMAT(///,' THE FOLLCWING TABLE CCMPARES DB OF FILTER TRANSFER FU
     1NCTION WITH FREQUENCY:',//,' STEP',10X,'FREC',10X'MAG(DB)',/)
      SF=1.0/ST
      WRITE(6,215) SF
  215 FORMAT(' SAMPLING FREQUENCY = ',E12.6,' HERZ FOR THIS TABLE.',/)
      DO 106 J=1,NPT
C
C EQUATION (33)
C
      ZEXP=CMPLX(CCS(OMEGA*ST),-SIN(OMEGA*ST))
      Y1=SCALE1-ZERO(1)*ZEXP
      Y2=SCALE1-PCLE(1)*ZEXP
      DO 9000 I=2,N
      Y1=Y1*(SCALE1-ZERO(I)*ZEXP)
      Y2=Y2*(SCALE1-POLE(I)*ZEXP)
 9000 CONTINUE
      YSUM(J)=CABS(Y1/Y2)
C
C COMPUTE MAGNITUDE OF FILTER TRANSFER FUNCTION IN DB
C
      DB(J)=20.*ALOG10(YSUM(J))
      F(J)=OMEGA/(2.*PI)
      WRITE(6,212) J,F(J),DB(J)
  212 FORMAT(2X,I3,10X,F6.1,7X,F10.5)
C
C INCREMENT THE STARTING FREQUENCY BY DELTA
C
```

```
      OMEGA=OMEGA+DELTA
  106 CONTINUE
C
C PLCT MAGNITUDE(CB) AS A FUNCTION CF FREQUENCY
C
      IF(LL.EQ.1) GC TO 30
      IF(LL.EQ.2) GC TO 31
      IF(LL.EC.3) GC TO 32
      IF(LL.EQ.4) GO TO 33
      IF(LL.EC.5) GC TO 34
   30 CALL DRAW(NPT,F,DB,0,0,LABEL1,ITITLE,0.,0.,C,0,C,C,8,8,1,LAST)
      GO TO 800
   31 CALL CRAW(NPT,F,DB,0,0,LABEL2,ITITLE,0.,0.,0,0,0,0,8,8,1,LAST)
      GO TO 800
   32 CALL CRAW(NPT,F,DB,0,0,LABEL3,ITITLE,0.,0.,C,0,C,0,8,8,1,LAST)
      GO TO 800
   33 CALL CRAW(NPT,F,DB,0,0,LABEL4,ITITLE,0.,0.,C,C,C,C,8,8,1,LAST)
      GO TO 800
   34 CALL CRAW(NPT,F,DB,0,0,LABEL5,ITITLE,0.,0.,C,0,C,C,8,8,1,LAST)
  800 LL=LL+1
      ST=ST/1.5
  105 CONTINUE
      END
0.99          0.90     0.85264   0.52250
0.99          0.90     0.85264  -0.52250
0.97          0.90     0.85941   0.51129
0.97          0.90     0.85941  -0.51129
0.95          0.90     0.86603   0.50000
0.95          0.90     0.86603  -0.50000
0.97          0.90     0.87250   0.48862
0.97          0.90     0.87250  -0.48862
0.99          0.90     0.87882   0.47716
0.99          0.90     0.87882  -0.47716
```

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13. ABSTRACT

A study of the physics of the piano reveals that while the upper partials of the steel strings are the eigen-frequencies of the complex tone, they are not integer multiples of the respective fundamentals. To properly measure and tune these eigen-partials, a digital filter capable of sweeping a major portion of the audio-frequency spectrum had to be implemented. Such a filter, a tuneable fixed-coefficient digital filter, is discussed as well as a simple pole-zero design procedure for determining the required coefficients. Each module, including the Frequency Deviation Detector and Counter, the Time-Base Generator, the Digital Filter, the Reference Frequency Generator and the Display and Control Module, of the proposed tuner is illustrated and discussed.

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